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DURABILITY METHODS DEVELOPMENT
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DURABILITY ANALYSIS METHODOLOGY DEVELOPMENT

J. N. YANG
THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, D.C.

S. D. MANNING
W. R. GARVER
GENERAL DYNAMICS CORPORATION
STRUCTURAL AND DESIGN DEPARTMENT
FORT WORTH DIVISION
P. O. BOX 748
FORT WORTH, TEXAS 76101

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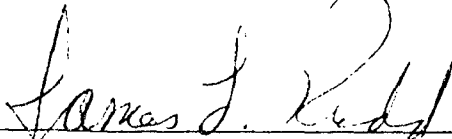AIR FORCE SYSTEMS COMMAND
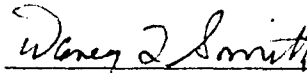WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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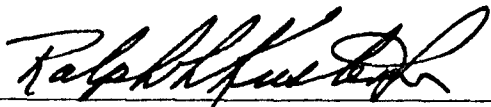This technical report has been reviewed and is approved for publication.

JAMES L. RUDD
Project Engineer

DAVEY L. SMITH
Structural Integrity Branch
Structures & Dynamics Division

FOR THE COMMANDER

RALPH L. KUSTER Jr., Colonel, USAF
Chief, Structures & Dynamics Division

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*George Washington University
Washington, D.C.

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This report describes the development of a statistical based durability analysis methodology for advanced metallic airframes. It can be used to analytically assure the U.S. Air Force's durability design requirements. Procedures and criteria are developed for analytically quantifying economic life, including procedures for evaluating the effects of inspection and repair maintenance on economic life and maintenance cost. A statistical model is developed for

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characterizing initial fatigue quality. The model combines the time-to-crack-initiation (TTCI) and the equivalent initial flaw size (EIFS) concepts into a statistical format with deterministic crack growth. Details of the methodology are presented, including derivations, analytical procedures, verification of the statistical model, and demonstration of the methodology capabilities/ potential. A verification of the durability analysis methodology for full-scale aircraft structure will be accomplished during Phase II of the program. It is shown that inspection and repair maintenance procedures have a limited effect on extending economic life.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

LIST OF SYMBOLS

a	=	Crack Size
a_1	=	Crack size below detection limit
a_2	=	Crack size beyond which the crack can be detected with certainty
a_{CR}	=	Critical crack size associated with the maximum design load P_{xx}.
$a(0)$	=	Flaw size at time $t=0$.
$a(t)$, $a(t_1)$, $a(t_2)$	=	Crack size at time t, t_1 and t_2, respectively
$a(\tau)$	=	Crack size at service interval τ
b, Q	=	**Crack growth constants in $\dfrac{da(t)}{dt} = Qa^b(t)$
b_i, Q_i	=	Crack growth constants in $\dfrac{da(t)}{dt} = Q_i a^{b_i}(t)$ for growing EIFS population for particular stress region
c	=	$b-1$
c_i	=	$b_i - 1$
C_1	=	Cost of inspecting one detail
$\bar{C}(I)$	=	Average maintenance cost, including inspections and repairs, in the service interval $(0, T_{I+1})$
$C_2(x)$	=	Cost of repairing one cracked detail having a crack size x
EIFS	=	Equivalent initial flaw size

**Appears in EIFS distribution – obtained from one data set of coupon specimens under the same loading spectra but a particular maximum stress level.

$f_{a(0)}(x)$ = Probability density function for EIFS

$f_{a(t)}(x)$ = Probability density function for crack size $a(t)$

$f_{a(T_j)}(x)$ = Probability density function for crack size at T_j

$f_{a(T_1)}(x)$, $f_{a(T_2)}(x)$ = Probability density function of the crack size $a(T_1)$ and $a(T_2)$, respectively, at the end of the noted service interval

$f_{a(T_1^+)}(x)$, $f_{a(T_2^+)}(x)$ = Probability density function of the crack size $a(T_1^+)$ and $a(T_2^+)$, respectively, right after the first and second inspection and repair

$f_{a(T_j+t)}(x)$ = Probability density function of the crack size $a(T_j+t)$ in the $j+1$ service interval

$f_{a(T_{j-1}+\tau_j)}(x)$ = Probability density function of the crack size $a(T_j)$ prior to the jth inspection

$F_{a(0)}(x)$ = Cumulative distribution for EIFS

$F_D(x)$ = Probability of detecting a crack size x

$F_D^*(x)$ = Probability of not detecting a crack size x , $1-F_D(x)$

$g_1(x)dx$ = Probability of detecting a crack in the size range $(x, x+dx)$ at $t=\tau_1=T_1$

$G(j)$ = Probability of detecting a crack of any size at the jth inspection

$G(1)$, $G(2)$, $G(3)$ = Probability of detecting a crack of any size at T_1, T_2 and T_3, respectively

I = Total number of scheduled inspections

$J_1(t)$ $= \dfrac{dy_1(t)}{dx} = \dfrac{dW(x,t)}{dx}$

$J_1(\tau_1)$ $= \dfrac{dy_1(\tau_1)}{dx} = \dfrac{dW(x,\tau_1)}{dx}$

$L(\tau), \bar{L}(\tau)$ $=$ Total and average number of details, respectively, in the entire component having a crack size $>x_1$ at any service time τ

m $=$ Exponent for function describing the probability of detecting a crack size x

NDI $=$ Nondestructive inspection

N_i $=$ Total number of details in the ith stress region

N^* $=$ Total number of details in the entire durability critical component

$N(i,\tau), \bar{N}(i,\tau)$ $=$ Total and average number of details, respectively, having a crack size exceeding x_1 at any service time τ

$p(i,\tau)$ $=$ Probability that a detail in the ith stress region will have a crack size $>x_1$ at the service time τ

$P[N(i,\tau) = n]$ $=$ Probability that the total number of details having a crack size exceeding x_1 in the ith stress region at the service time τ, $N(i,\tau)$, is equal to n

$P_f(\tau), P_f(T_j)$ $=$ Probability of failure of the entire component in the service interval $(0,\tau)$ and $(0,T_j)$, respectively

$P_i(0,\tau)$ $=$ Probability of failure of one detail in the ith stress region of the component within the service interval $(0,\tau)$

$P_i(T_{j-1},T_j)$ $=$ Probability of failure of a detail in the ith stress region within the jth service interval (T_{j-1},T_j)

$q_1(z_1,z_2)$, $q_2(z_1,z_2)$ = Probability of detecting a crack in the size range z_1 and z_2 during the first and second inspection, respectively

$q_j(z_1,z_2)$ = Probability of detecting (or repairing) a crack in the size range (z_1,z_2) at the jth inspection, i.e., T_j (j=1, 2, ...)

$R(\tau)$, $R(T_j)$ = Probability of survival of the entire durability critical component in the service interval $(0,\tau)$ and $(0,T_j)$, respectively

t, t_1, t_2 = Flight hours at time t, t_1 and t_2, respectively

T_i = Cumulative service hours at time of inspection and repair maintenance (i=0, 1, 2, ...)

TTCI = Time to crack initiation

W = A general function representing the crack growth master curve

x, x_1 = Crack size

x_u = Upper bound limit for EIFS

$y_1(\tau)$, $y_1(t)$, $y_{CR}(\tau)$ = A value of $a(0)$ corresponding to a value x of $a(\tau)$, $a(t)$ and $a_{CR}(\tau)$, respectively

α, β, ϵ = Weibull distribution parameters for shape, scale and position, respectively.

$\Delta a(t_j)$ = Crack growth increment per flight hour at time t_j

σ_{max} = Maximum average stress on gross cross section (ksi)

$\sigma_N^2(i,\tau)$, $\sigma_L^2(i,\tau)$ = Variance of $N(i,\tau)$ and $L(\tau)$, respectively

τ_i = Service interval between scheduled inspection and repair maintenance periods (i = 1, 2, 3,...)

$\Gamma(\ )$ = Gamma function

DEFINITION OF TERMS

1. <u>Deterministic Crack Growth</u> - Crack growth parameters are treated as deterministic values resulting in a single value prediction for crack length.

2. <u>Durability</u> - "The ability of the airframe to resist cracking (including stress corrosion and hydrogen induced cracking), corrosion, thermal degradation, delamination, wear, and the effects of foreign object damage for a specified period of time" [1].

3. <u>Durability Analysis</u> - An analysis for quantifying the extent of structural damage as a function of service hours used for ensuring compliance with the Air Force's durability design requirements for advanced metallic airframes.

4. <u>Economic Life</u> - "... occurrence of widespread damage which is uneconomical to repair and, if not repaired, could cause functional problems affecting operational readiness" [1].

5. <u>Economic Life Criteria</u> - Analytical guidelines for quantifying economic life.

6. <u>Economic Repair Limit</u> - Maximum damage size that can be economically repaired (e.g., repair 0.030"-.050" radial crack in fastener holes by reaming hole to next size).

7. <u>Equivalent Initial Flaw Size (EIFS)</u> - A hypothetical crack assumed to exist in the structure prior to service. It characterizes the equivalent effect of actual initial flaws in a structure detail. Values are usually determined by back extrapolating fractographic results.

8. <u>Initial Quality</u> - "A measure of the condition of the airframe relative to flaws, defects, or other discrepancies in the basic materials as introduced during manufacture of the airframe" [1].

9. <u>Probability of Crack Exceedance</u> - Refers to the probability of exceeding a specified crack size at a given service time. It can be determined from the statistical distribution of crack sizes and it is the fundamental quantity for predicting economic life.

10. <u>Time-To-Crack-Initiation (TTCI)</u> - The time or service hours required to initiate a specified (observable) crack size in a structural detail.

SECTION I

INTRODUCTION

Current U. S. Air Force structural integrity and durability design specifications [1-3] require that airframe components be designed such that the economic life be analytically predicted. The conventional fatigue analysis, while capable of estimating design life, does not lend itself to predicting the economic life, nor is it capable of providing a definition of economic life. In predicting the economic life, the entire crack population should be taken into account, and hence the statistical approach is essential. Literature describing the current practice regarding crack growth damage accumulation with applications to structural safety, durability, damage tolerance, and reliability is available [4-9] . In particular, the durability and the economic life requirements have been reviewed in detail in Ref. 9.

The cost of maintenance, including the costs of inspection, repair, rework, replacement, down time, etc., for aircraft structures, in order to fulfill the requirements of safety, durability, damage tolerance and reliability, is of practical importance. It is well-known that after a certain service life, referred to as the economic life, either the cost of maintenance or the number of cracks exceeding the economical repair crack size increases so rapidly that the durability requirement cannot be satisfied. In an attempt to demonstrate statistically the existence of the economic life quantitatively, an exploratory investigation was made to estimate the service cracks and maintenance cost [10]. In Ref. 10, however, restrictive limitations were imposed, for simplicity, in the exploratory studies.

In this volume, an analytical durability methodology that is capable of quantitatively predicting the economic life of advanced metallic aircraft structures has been developed. The methodology is based on sound analytical and statistical approaches. It accounts for various service conditions, such as any type of initial fatigue quality, crack growth damage accumulation, loading spectra, material/structural properties, usage change, inspection and repair maintenance, etc. Hence, the methodology presented herein is general enough for practical applications. The formulation allows for the determination of the economic life using either one of the following two criteria; (i) a rapid increase of the number of crack damages exceeding the economic repair crack size, and (ii) rapid increase of the maintenance cost. The economic repair crack size a_e is

1

defined as the crack size below which the least expensive repair procedure can be used, such as remaing the fastener holes to the next hole size. The size a_e is usually between 0.03" and 0.05" depending on the location and the fastener hole size.

The durability critical component may also be subjected to a scheduled (nonperiodic) inspection and repair maintenance procedure as shown in Fig. (1), in which a component without a maintenance procedure is a special case. Within the framework of the first criterion of the economic life, the percentage (or numbers) of cracks exceeding a_e is obtained as a function of the service time. The percentage of cracks exceeding a_e for both cases, with or without a scheduled maintenance, is schematically shown in Fig. (2). For the second criterion of the economic life, the average cost of maintenance, including the cost of inspection and repair, has been formulated as a function of service time, thus permitting a determination of the economic life.

The analytical methodology is demonstrated herein by a numerical example. A durability critical component of a fighter aircraft under scheduled inspection and repair maintenance procedures has been considered to demonstrate the methodology developed herein. The number of cracks in the component exceeding the economic repair crack size with any probability and confidence levels is obtained as a function of service time. It is shown numerically that the number of cracks exceeding the economic repair crack size (with any probability and confidence levels) increases rapidly after a certain service time, thus determining the possible economic life of the durability critical component. While the inspection and repair maintenance procedures have a significant impact on the safety and reliability of aircraft structures [e.g., 11-17], its effect on the economic life of a component is shown to be limited.
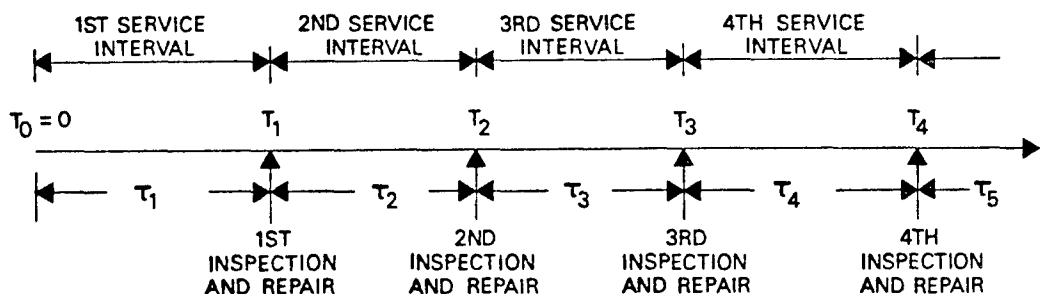
Figure 1   Inspection and Repair Maintenance Schedule

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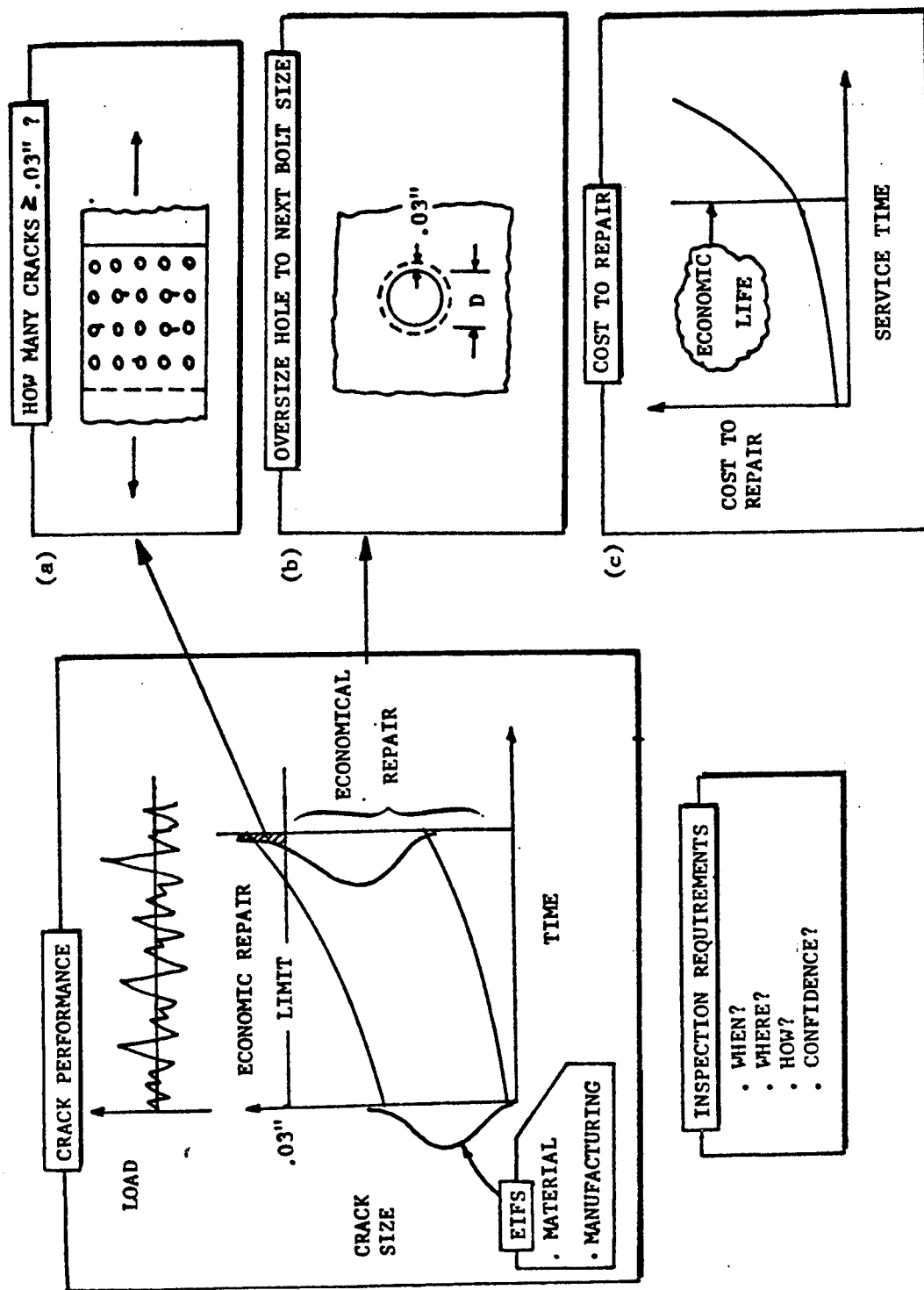
Figure 2 Durability Analysis Elements

SECTION II

TECHNICAL OVERVIEW

An important input parameter to the durability analysis is the initial fatigue quality of the critical parts of the aircraft structures. For engineering analysis and design purposes, the initial fatigue quality has been characterized by either the time to crack initiation (TTCI) or the equivalent initial flaw size (EIFS), and attempts have been made in the literature [6, 18-35] to characterize the statistical distributions of both using available laboratory or field data.

The time to crack initiation is defined as the time at which a visible crack size a_0, say 0.03", occurs. In the present development of the durability analysis methodology, the input initial quality is chosen to be the time to crack initiation (TTCI). The reasons are: (i) the crack size a_0 at crack initiation is physically observable, and (ii) test data can be obtained from the coupon specimens and the full-scale components, including the test data of fractography.

The time to crack initiation, however, is a function of so many variables that if any one variable changes, such as loading spectra or stress level, test data may have to be generated again, since the functional relation between TTCI and other influencing variables is not available to date. This situation becomes particularly critical in the present durability analysis where the crack growth damage at each location of the entire durability component has to be estimated. Since the maximum stress level of the loading spectra may vary from one location to another, it may not be economically practical to conduct laboratory tests for TTCI for all maximum stress levels which may occur at all the fastener holes.

As a result, the input data of TTCI, which is statistically characterized by the three-parameter Weibull distribution [20, 21], is converted (transformed) through backward extrapolation herein by use of a crack propagation law to determine the statistical distribution of the "equivalent" initial flaw size [20,21,36] expediently. Such an approach is motivated by the expedient necessity stated above. Various questions associated with such an approach will be justified later, except to emphasize that the equivalent initial flaw sizes thus obtained for one type of particular aircraft cannot necessarily be used for other types of aircraft with completely different mission characteristics. Note that the statistical distribution of the equivalent initial flaw size derived from the three-parameter Weibull distribution of TTCI is consistent with the physical fatigue wear-out process of metals [20,21,36].

5

It is, therefore, very important to emphasize that the durability analysis methodology developed herein does not distinguish the time to crack initiation approach from that of the equivalent initial flaw size approach. This is because both approaches are interrelated. Again, it should be noted that one set of the equivalent initial flaw size data obtained from one type of aircraft cannot necessarily be used for other types of aircraft with different mission characteristics.

The entire population of the equivalent initial flaw size, as specified by its statistical distribution derived from TTCI data, is then subjected to service loading spectra consisting of possibly various different missions. The crack growth damage accumulation under spectrum loading is computed using a general crack growth master curve which can be obtained either by a general computer code or by any crack growth model or experimental results, including those of fractography. It should be mentioned that the crack growth damage calculation approach recently proposed by Gallagher [37-39] can also be used conveniently.

Thus, the statistical distribution of crack sizes at any service time τ can be obtained from that of the equivalent initial flaw size distribution through the transformation of the crack growth damage accumulation master curve. Then, the percentage of cracks exceeding the economic repair crack size a_e with certain probability and confidence levels can be computed as shown in Fig. (2).

A maintenance program can be implemented in service where inspections and repairs are performed at scheduled intervals. The statistical uncertainties of the NDI procedure in crack detection has been accounted for. When a cracked detail is detected and repaired, its fatigue strength is assumed to be renewed in the sense that its crack size distribution is identical to that of the equivalent initial flaw size prior to service, and its crack propagation characteristic remains unchanged, i.e., neglecting the possible effects of cold-work and others. The crack growth damage accumulation of the repaired details, referred to as the renewal details, has also been taken into account.

During inspection, however, some cracked details may not be detected depending on the resolution capability of a particular NDI procedure employed. When a crack is missed during inspection, it continues to grow until the next inspection, at which time it will have a higher probability of being detected. However, the probability of failure of the component increases. The reliability of the entire durability critical component in any service interval has been formulated analytically.

As a result, the statistical distribution of crack sizes is shifted and changed continuously in service due to crack propagation. It is also subjected to truncation and modification after each inspection and repair.

The statistical distribution of crack sizes at any service time is derived, and the number of cracked details exceeding any crack size with certain probability and confidence levels are obtained as a function of service time.

Finally, the average number of cracks detected and repaired during each maintenance is obtained and the average cost of maintenance is formulated as a function of the service life.

S E C T I O N I I I

STATISTICAL CHARACTERIZATION OF INITIAL FATIGUE QUALITY

One of the most important factors affecting the durability and fatigue life of a structural component is the initial fatigue quality. The initial fatigue quality of a component depends on such variables as material properties, drilling techniques, welding qualities, manufacturing processes, assembling of structures, etc. To characterize the initial fatigue quality as a function of various influencing variables is a major undertaking. Nevertheless, for analysis and design purposes, the initial fatigue quality can be characterized by either the time to crack initiation (TTCI) or the equivalent initial flaw size (EIFS).

The time to fatigue crack initiation can be defined as the time at which a visible crack size a_0, say 0.03", occurs. For practical engineering purposes, attempts have been made to characterize the statistical distribution of time to fatigue crack initiation for applications to fatigue analysis and design of aircraft structures [22-34].

The advantages of the time to crack initiation (TTCI) approach are (i) the crack size at crack initiation is physically observable, (ii) test data can be obtained from coupon specimens and full scale components directly, and (iii) linear fracture mechanics for fatigue crack propagation can be applied after crack initiation. Hence, the fatigue process of metals has been traditionally divided into three stages: (a) fatigue crack initiation, (b) fatigue crack propagation (stable crack propagation), and (c) fatigue fracture (unstable crack propagation).

The disadvantage, however, is that it is a function of several variables, such as stress level, loading spectra, manufacturing processes, etc. Thus, if any one of the influencing variables changes, e.g., stress level or loading spectra, test data may have to be generated again, since the functional relation between TTCI and these influencing variables has not been established to date. This situation becomes especially critical in the durability analysis of aircraft structures where the crack growth damage at each location of the critical component has to be estimated. Since the maximum stress level of the loading spectrum may vary from one location to another, it is economically impractical to conduct laboratory tests for TTCI for each maximum stress level.

Another characterization of the initial fatigue quality is the initial flaw size existing in the structure prior to service [6,18,19,35]. Detectable flaws or defects do exist in the components prior to service [16,10]. However, most of the "initial flaws" existing in the structure are not detectable by any NDI technique. Thus, the initial flaw size is usually obtained from the results of laboratory fatigue tests. When detectable flaws are observed during fatigue tests, they are extrapolated backwards using a suitable crack propagation law to estimate their "equivalent" initial flaw size (EIFS) [6,18,19,35]. The backward extrapolation has to be validated using fractographic results. This approach implies that the entire fatigue process is essentially subcritical flaw growth. Test data of the equivalent initial flaw size thus obtained have been available, and their distributions have been characterized by the four-parameter Johnson distribution [6,10,18,19,35].

The advantage of this approach is that, once the initial flaw size distribution is established, the fatigue crack propagation under various service loading spectra can be predicted analytically without further experimental tests. Furthermore, the equivalent initial flaw size approach can be used to study the effect of various factors, such as drilling techniques, manufacturing processes, etc., on the fastener hole quality [35].

The input initial fatigue quality to the durability analysis methodology developed herein is the time to crack initiation, because of the advantages stated above and because of the familiarity of practicing engineers. TTCI data should be obtained from the test results under design loading spectra. Since TTCI is a function of several variables, in particular the stress level in the loading spectra, and since the stress level varies from one region of the durability critical component to another, it may be economically impractical to generate test results of TTCI for each stress level. This is critical to the durability analysis in which the entire crack population in the whole component should be taken into account.

As a result, the input TTCI data under the design loading spectra with one maximum stress level is used in the analysis. Then, the TTCI data is proposed to be fitted by the three-parameter Weibull distribution [20,21]. The statistical distribution of the equivalent initial flaw size is derived from the three-parameter Weibull distribution of TTCI and the durability analysis is started from the statistical distribution of the EIFS. This expedient approach simplifies significantly the durability analysis procedures [20] as will be discussed later along with the justification of such an approach.

Since the Weibull distribution for TTCI represents the wear-out fatigue process, the derived new distribution for the equivalent initial flaw size bears the physical notion of the wear-out fatigue process. Methods of estimating the distribution parameters and confidence levels will be described. Finally, test results of TTCI and EIFS are presented to demonstrate the validity of the proposed distribution. It will be shown that the derived distribution fits the EIFS data better than any other distribution function.

3.1 Time to Crack Initiation (TTCI)

Traditionally, the lognormal distribution has been used to describe the statistical distribution of the time to fatigue crack initiation (TTCI) [22-23]. However, the failure rate (or risk function) associated with the lognormal distribution initially increases and then eventually decreases. This is inconsistent with the fatigue wear-out model. From a physical standpoint, it is generally agreed that metal fatigue is a wear-out process. For example, the longer a specimen has survived fatigue testing, the probability of specimen failure increases. Thus, the fatigue life failure rate should increase monotonically. If the failure rate of the fatigue life is a positive power function of the service time, then it can be derived mathematically that the distribution function of the time to crack initiation is Weibull, i.e.,

$$F_T(t) = P[T \leq t] = 1 - e^{-(t/\beta)^{\alpha}}, \quad t \geq 0 \tag{1}$$

in which T is a random variable indicating the time to crack initiation, α is the shape parameter and β is the scale parameter. In Eq. (1), $F_T(t) = P[T \leq t]$ is the probability that the time to crack initiation T is smaller than or equal to a value t.

The Weibull distribution given by Eq. (1) has been used to describe the time to crack initiation [24-34], and methods of estimating the parameters α and β from test data are available [26-27].

Observations of extensive specimen test data indicate that α is fairly constant for a particular material and it is not sensitive to specimen geometry, testing method, specimen size, etc. Compilation of coupon test data indicates that $\alpha = 4.0$ may be appropriate for aluminum [27]. Moreover, attempts have been made to apply the Weibull distribution for the time to crack initiation to available service data for various types of aircraft, for instance C-130, C-141, F4, etc. [30-34]. An improvement in predicting the time to service crack initiation has also been made by taking into account the statistical variability of service loads [34].

11

The lower bound of the two-parameter Weibull distribution given by Eq. (1) is zero. The lower bound, in effect, depends on the definition of the crack size a_0 at crack initiation. As a result, the following three-parameter Weibull distribution should be used for generality [20],

$$F_T(t) = P[T<t] = 1 - \exp \left\{ - \left(\frac{t-\varepsilon}{\beta} \right)^\alpha \right\} \ ; \ t \geq \varepsilon \tag{2}$$

in which ε is the lower bound of TTCI. The lower bound ε increases as the defined crack size a_0 at crack initiation increases.

3.2 Fatigue Crack Propagation in Small Crack Size Region

As mentioned previously, the equivalent initial flaw sizes (EIFS) are not observable by any NDI technique. They are obtained from detectable crack sizes observed during fatigue tests by backward extrapolation. Such an extrapolation must be verified using fractographic results. A significant contribution to the prediction of crack growth damage under complex flight loading has been made recently by Gallagher who indicates that the crack growth rate under flight-by-flight loading spectra can be written as follows [37-39].

$$\frac{da}{dt} = G(\overline{K}_{max})^\lambda \tag{3}$$

in which t represents the flight (or flight hour or miniblock of spectra), G and λ are parameters depending on such factors as loading spectra, material and structural properties, etc.

In Eq. (3), $\overline{K}_{max}$ is the root mean square of the maximum stress intensity factor,

$$\overline{K}_{max} = \left(\overline{\sigma^2_{max}} \right)^{1/2} \cdot \psi(a) \tag{4}$$

where $\overline{\sigma^2_{max}}$ is the mean square of the maximum stress in the flight spectra and $\psi(a)$ can be expressed in terms of the stress intensity factor coefficient $\beta(a)$, as

$$\psi(a) = \beta(a)\sqrt{\pi a} \qquad\qquad (5)$$

The stress intensity factor coefficient $\beta(a)$ depends on the crack size, the specimen width, the crack geometry (e.g., through-the-thickness crack, part-through crack, corner crack), etc. In general, $\psi(a)$ can be approximated by a power series of the crack size a as

$$\psi(a) = \sum_{i=1}^{\infty} n_i a^{m_i} \qquad\qquad (6)$$

In the region of small crack size, i.e., smaller than the crack size a_0 at crack initiation, a good approximation can be obtained by taking the first term only, i.e.,

$$\psi(a) \cong n_1 a^{m_1} \qquad\qquad (7)$$

Substituting Eqs. (4) and (7) into Eq. (3), one obtains

$$\frac{da(t)}{dt} = Q a^b(t) \qquad\qquad (8)$$

The advantage of the miniblock approach proposed by Gallagher [37-39] is that the crack growth damage can be computed by performing the flight-by-flight integration rather than the cycle-by-cycle integration, thus reducing a significant amount of computer time.

While the validity of the miniblock approach has been verified [37-39], Eq. (8) needs to be verified using physical observations.

If plots are obtained of log da/dt versus log a for all specimens tested in a data set, then a curve fitting technique can be used to find the best-fitting straight line through the data points. A least squares technique is sufficient and by minimizing the least squares error, "composite" values of b and Q, the slope and intercept of the line, respectively, can be determined for the data set as a whole. Typical plots of test data and Eq. (8) demonstrate good agreement as shown in Figs. (3) and (4). The relation given by Eq. (8) eventually becomes invalid when the crack size becomes large as will be discussed later.
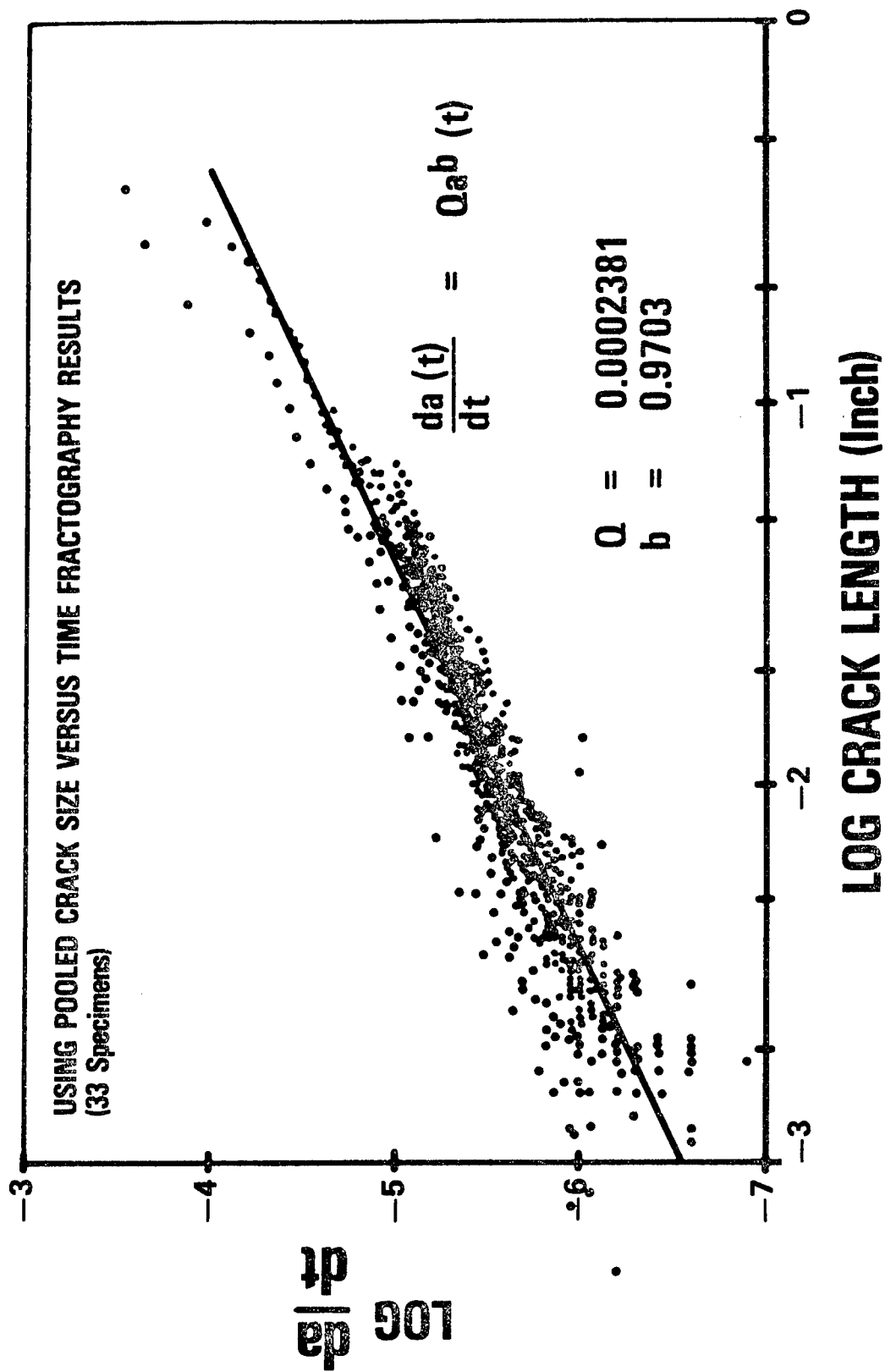
Figure 3   Log Crack Length Versus Log da/dt for WPF (33) Data Set

14
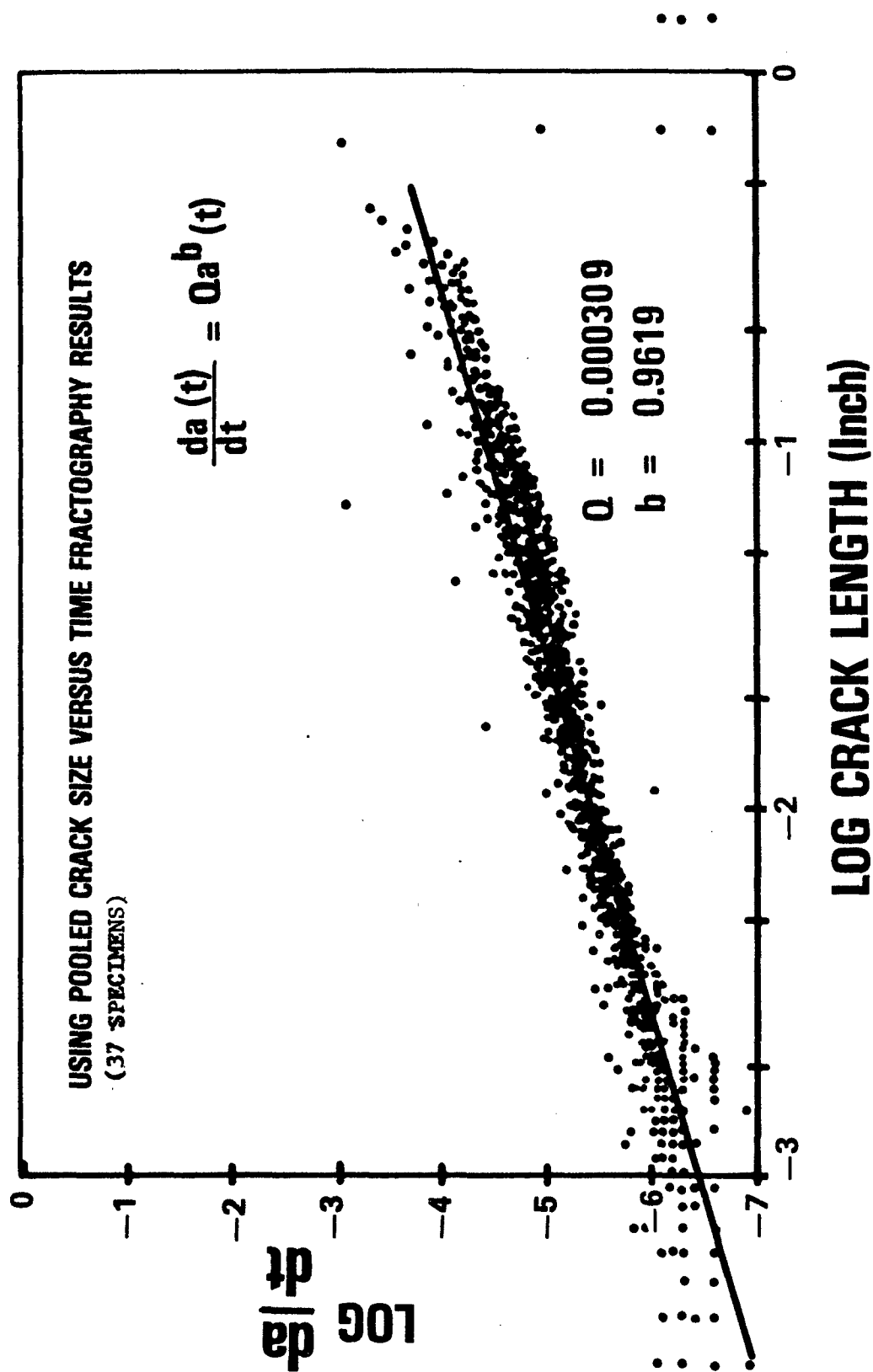
Figure 4 Log Crack Length Versus Log da/dt for XWPF (37) Data Set

15

By back extrapolation from the observed TTCI values using Eq. (8) and these composite Q & b values, the EIFS distribution can be found. Although this distribution cannot be verified by direct physical observation, when grown out to some time t again using Eq. (8) , the predicted distribution of crack sizes at this time t compares very closely with the observed distribution. This suggests that the crack growth model is valid and that the composite Q and b values obtained represent an average growing crack sufficiently well. These findings are detailed in Section 3.5.

3.3 Statistical Distribution of EIFS

Let $a(T)$ be the crack size at T flight hours, where T is the time to crack initiation, and $a(0)$ be the initial crack size (at t=0) prior to service. It is obvious that $a(T)=a_0$ is the crack size at crack initiation. Integration of Eq. (8) from t=0 to t=T yields the relation between the initial flaw size $a(0)$ and a_0,

$$\text{EIFS} = a(0) = \frac{a_0}{\left(1+a_0^c \, cQT\right)^{1/c}}\tag{9}$$

in which

$$c = b - 1\tag{10}$$

The statistical distribution of time to crack initiation T, is given by Eq. (2) . Thus, the statistical distribution of the initial flaw size, $a(0)$, can be derived from Eq. (2) through the transformation of Eq. (9) as follows:

$$F_{a(0)}(x) = P\left[a(0)\leq x\right] = P\left[\frac{a_0}{\left(1+a_0^c \, cQT\right)^{1/c}} \leq x\right]$$

$$= P\left[a_0^c \leq x^c(1+a_0^c cQT)\right] = P\left[T\geq \frac{1}{cQ}(x^{-c}-a_0^{-c})\right]$$

$$= 1 - P\left[T\leq \frac{1}{cQ}(x^{-c}-a_0^{-c})\right] \tag{11}$$

Substituting Eq. (2) into Eq. (11) , one obtains the distribution function $F_{a(0)}(x)$ of the initial crack size as follows

$$F_{a(0)}(x) = \exp\left\{-\left[\frac{x^{-c}-a_0^{-c}-cQ\epsilon}{cQ\beta}\right]^{\alpha}\right\} \quad ; \ x \leq x_u \qquad (12)$$

$$= 1; \qquad\qquad x \geq x_u$$

where x_u is the upper bound of the initial crack size.

$$x_u = \left(a_0^{-c}+cQ\epsilon\right)^{-1/c} \qquad (13)$$

Such a transformation is schematically shown in Fig. (5).

The EIFS cumulative distribution, $F_{a(0)}(x)$, Eq. (12), cannot be verified directly from fractographic results. However, the cumulative distribution, $F_{a(t)}(x)$, of the crack size $a(t)$ at any service time t where crack sizes are observable from fractographic results can be derived from $F_{a(0)}(x)$. A verification of $F_{a(t)}(x)$ then gives an indirect verification of $F_{a(0)}(x)$. $F_{a(t)}(x)$ is verified using actual fractography in Section 3.5.

The cumulative distribution of crack sizes at any service time t, $F_{a(t)}(x)$, is described by Eqs. (14) and (15) (**See** Appendix for derivation),

$$F_{a(t)}(x) = \exp\left\{-\left[\frac{y_1^{-c}(t)-a_0^{-c}-cQ\epsilon}{cQ\beta}\right]^{\alpha}\right\} \quad ; \ 0 \leq y_1(t) \leq x_u \qquad (14)$$

$$= 1 \ ; \quad y_1(t) \geq x_u$$

where,

$$y_1(t) = \frac{x}{[1+x^c cQt]^{1/c}} \qquad (15)$$

3.4 Estimation of Parameters and Confidence Level

The three parameters, α, β, and ϵ, appearing in Eq. (2) should be determined from test results of time to crack initiation. The curve fitting procedure along with the least square method can be employed to estimate α, β, and ϵ as follows.

17
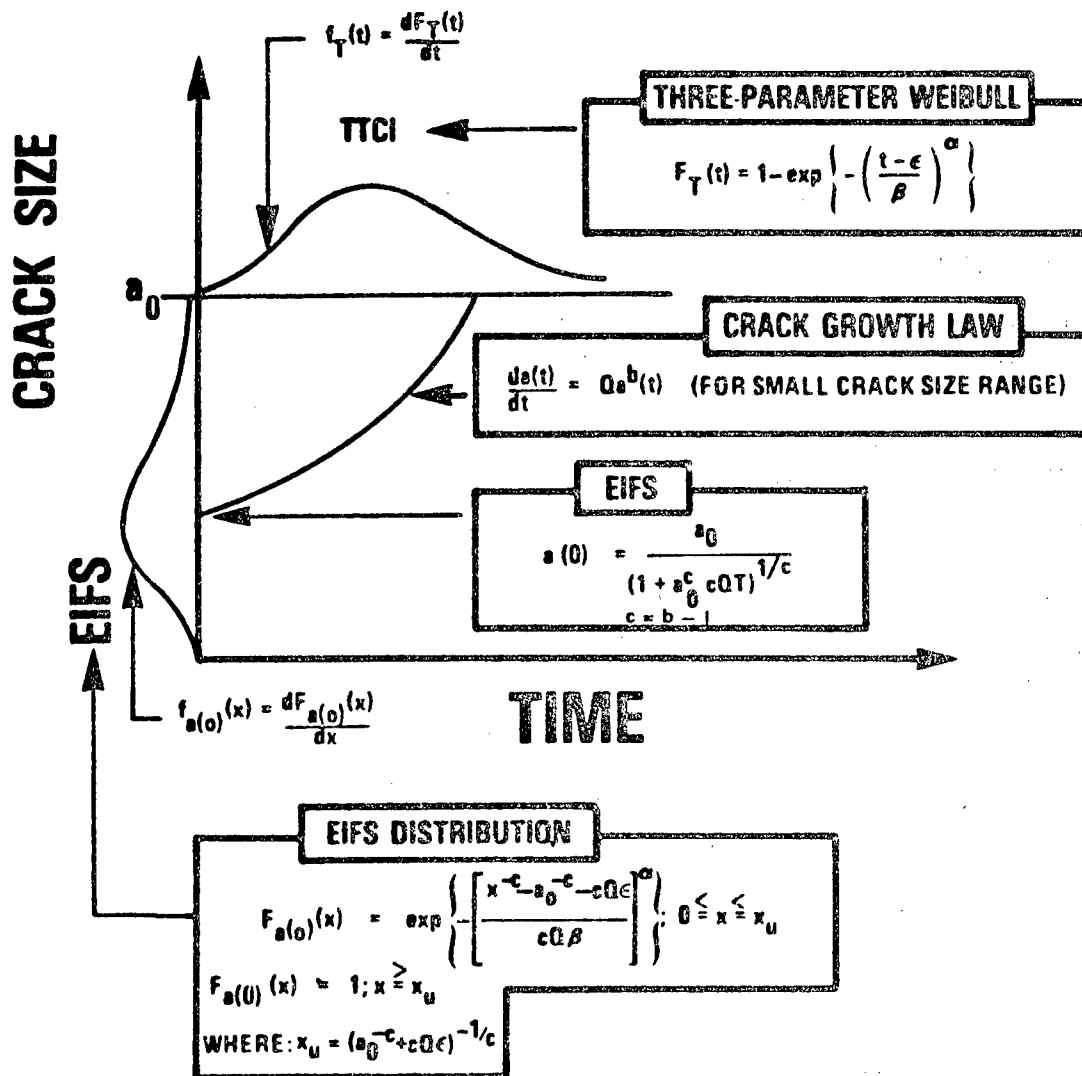
Figure 5 Elements of Initial Fatigue Quality Model

First, a value is assumed for the location parameter ε. Then, Eq. (2) is transformed into the following

$$\ell n[-\ell n(1-F_T(t))] = \alpha \ell n(t-\varepsilon) - \alpha \ell n \beta \qquad (16)$$

Let

$$Z = \ell n[-\ell n(1-F_T(t))]$$

$$Y = \ell n(t-\varepsilon) \qquad \qquad (17)$$

$$U = -\alpha \ell n \beta$$

Then, Eq. (16) reduces to a straight line in the (Y,Z) plane, referred to as the probability paper,

$$Z = \alpha Y + U \qquad (18)$$

Test data of time to crack initiation can be plotted on the probability paper, and α and β are estimated using the least square fit.

Another value of ε is then assumed and the same procedure is repeated to estimate another set of α and β. The final values of α, β, and ε are chosen to minimize the corresponding mean square error between the test data and the fitted distribution.

For given α and ε, the γ confidence level for β, denoted by β_γ, is given by [27]

$$\beta_\gamma = \beta \left[\frac{1}{2n} \kappa_\gamma^2(2n) \right]^{-1/\alpha} \qquad (19)$$

in which n is the total number of test data points and $\kappa_\gamma^2(2n)$ is the γ-fractile of the chi-square variate with 2n degrees of freedom. Eq. (19) would be exact if the estimate of β were obtained using the maximum-likelihood method.

3.5 Test Results And Correlation

Test results from the "Fastener Hole Quality" program [35] will be used to verify the EIFS distribution, Eq. (12) . Test results for two types of Aluminum (7475-T7351) specimens subjected to a F-16 fighter 400-hour

block spectrum with a maximum gross stress of 34 ksi are used: (1) WPF (no load transfer) and (2) XWPF (15% load transfer). Specimen details are shown in Fig. (6) . Observed TTCI's at a crack size of 0.03" along with crack sizes at two different times for WPF and XWPF data sets are presented in Tables 1 and 2, respectively.

The WPF (38) data set (Table 1) includes five specimens with only one fractographic observation/specimen (i.e., one crack size and one TTCI). Crack sizes and TTCI's were computed for these five WPF specimens (Table 1) using Eq. (8) and the crack growth model constants, Q and b (Table 3), based on the WPF (33) data set. The WPF (33) data set includes all specimens in Table 1 with observed TTCI's and applicable crack sizes. Both data sets, WPF (38) and WPF (33), are used later.

The EIFS cumulative distribution will be validated as follows: First, it will be shown that the three-parameter Weibull distribution fits the observed TTCI results for a_0=0.03". Second, the theoretical cumulative EIFS distribution will be compared against ranked EIFS predictions (t=0). Third, the predicted cumulative crack size distributions for two different times and two different specimens will be compared against the corresponding ranked crack sizes observed.

Weibull best fit plots for WPF (38), WPF(33), and XWPF data sets are shown in Figs. (7) through (9) , respectively. The three-parameter Weibull distribution fits the observed TTCI results well. Parameter values used are summarized in Table 3.

The values of α, β and ϵ from the test results of TTCI (Tables 1 and 2) and the values of b and Q based on crack growth fractography data (Figs.(3) and (4)), can now be used to examine the correlation between the observed and the theoretically predicted results.

EIFS values are computed for three data sets, WPF (38), WPF (33) and XWPF, using the TTCI results of Tables 1 and 2 and Eq. (9) . Results are ranked and plotted in Figs. (10)through (12) as circles. Predicted cumulative distribution curves, represented by Eq. (12) , are plotted in Figs.(10) through (12) . Furthermore, observed crack sizes at two times, presented in Tables 1 and 2, are also ranked and plotted in Figs. (10) through (12) and compared with the predicted cumulative distribution represented by Eq. (14) for WPF (38), WPF (33), and XWPF data sets. The correlation between the observed and the predicted results is excellent.

NO LOAD TRANSFER SPECIMEN
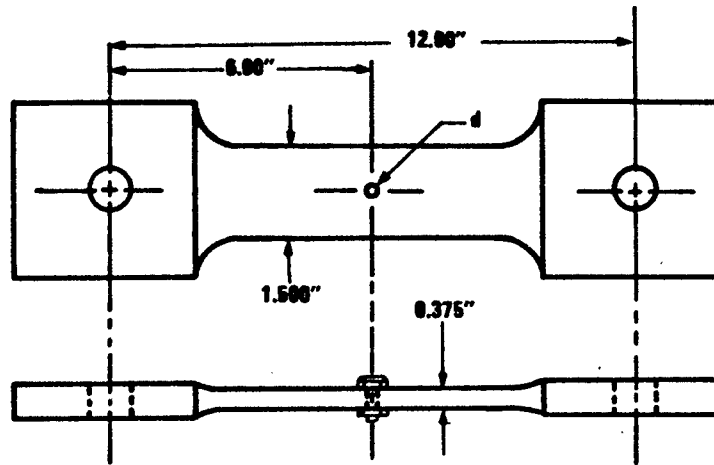
d = 0.250"

15% LOAD TRANSFER SPECIMEN
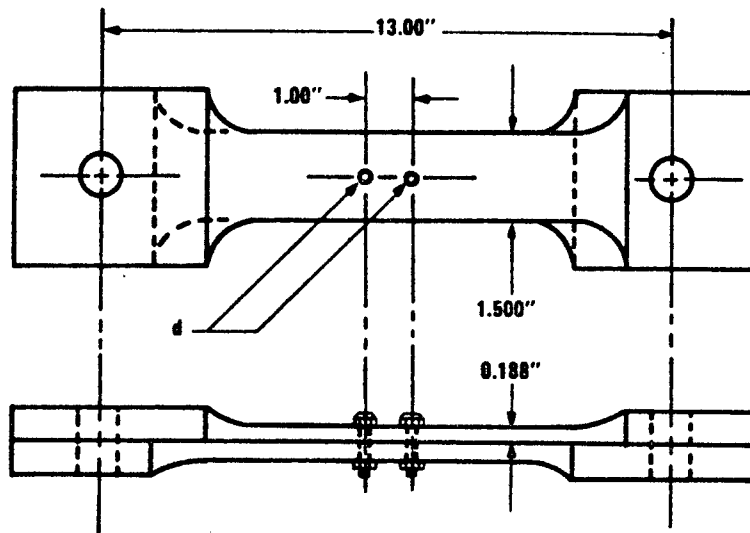
d = 0.250"

Figure 6   Test Specimen Details

21

Table 1 Observed Test Results For WPF Data Set

SPECIMEN NO. [1]	WPF (38) [3]		
	TTCI [2] (HOURS)	a(t = 9200 Hrs) (Inch)	a(t = 14800 Hrs) (Inch)
WPF −11	6563	0.0617	0.2570
− 7	9312	0.0287	0.3557
−16	10629	0.0229	0.0582
−21	10892	0.0205	0.0654
− 9	11357	0.0185	0.0570
− 6	11637	0.0142	0.0687
− 8	12053	0.0143	0.0548
−18	13379	0.0118	0.0387
−23	13544	0.0118	0.0379
−32	13762	0.0099	0.0372
−14	13773	0.0112	0.0357
−17*	(13783)	(0.0067)	(0.0392)
−25	13939	0.0082	0.0360
−35	14098	0.0072	0.0354
−13	14123	0.0082	0.0348
−12	14149	0.0064	0.0361
−19	14252	0.0073	0.0332
−22	14350	0.0082	0.0329
−10	14400	0.0082	0.0322
−34	14436	0.0080	0.0322
−29	15499	0.0056	0.0259
−24	15600	0.0049	0.0252
−33	15799	0.0038	0.0225
− 20	16128	0.0034	0.0217
−40	16141	0.0047	0.0215
−31	17109	0.0058	0.0185
−37	17134	0.0039	0.0172
−39	17185	0.0048	0.0170
−38	17507	0.0052	0.0162
−36	17620	0.0032	0.0151
−26	17639	0.0037	0.0148
−30	18066	0.0037	0.0128
−27	18357	0.0028	0.0121
−41*	(18696)	(0.0022)	(0.0105)
−28	19154	0.0031	0.0099
−43*	(20140)	0.0015	0.0071
−15*	(20449)	(0.0013)	(0.0065)
−42*	(23767)	(0.0005)	(0.0026)

NOTES

[1] Ref. Fig. 6

[2] For $a_0 = 0.03''$

[3] WPF(38) = total WPF data set

* Single observation/specimen (i.e., single data point: a_i, t_i)
  a_i = crack size

  t_i = time

(xxx) = Value derived from single observation using Eq. 8 and results in
        Table 3 for WPF(33)

22

Table 2 Observed Test Results For XWPF Data Set

SPECIMEN NO. ①	XWPF (Ref. 35)		
	TTCI ② (HOURS)	a(t = 7200 Hrs) (Inch)	a(t = 10800 Hrs) (Inch)
XWPF-36	6106	0.0392	0.0814
-38	6147	0.0418	0.0922
-18	7457	0.0264	0.0943
- 9	7545	0.0248	0.0940
- 7	7721	0.0211	0.1098
- 3	8425	0.0186	0.0910
-34	8528	0.0153	0.0753
-31	8636	0.0195	0.0525
-29	8908	0.0206	0.0429
-15	8968	0.0140	0.0582
- 8	9078	0.0168	0.0487
-22	9085	0.0131	0.0575
- 1	9316	0.0195	0.0386
- 2	9973	0.0111	0.0402
-11	10253	0.0067	0.0363
-12	10457	0.0081	0.0333
-16	10908	0.0088	0.0288
-24	11045	0.0120	0.0281
-19	11051	0.0049	0.0271
-30	11071	0.0052	0.0257
- 5	11370	0.0087	0.0264
-23	11493	0.0042	0.0201
-14	11564	0.0022	0.0110
-28	11571	0.0047	0.0213
-17	11698	0.0063	0.0223
-27	11708	0.0058	0.0210
-37	11920	0.0048	0.0215
-33	12051	0.0071	0.0203
-13	12062	0.0045	0.0174
- 6	12118	0.0061	0.0208
-21	12302	0.0060	0.0191
-25	12389	0.0070	0.0193
-26	12505	0.0040	0.0140
-35	12629	0.0045	0.0169
- 4	12655	0.0041	0.0136
-10	13339	0.0030	0.0100
-20	16008	0.0007	0.0034

NOTES: ① Ref. Fig. 6
② $a_0 = 0.03''$

Table 3 Model Parameters Based On Observed Test Results

Parameter	Specimen		
	WPF(38)	WPF(33)	XWPF(37)
b	0.9703	0.9703	0.9620
c	-0.0297	-0.0297	-0.0380
Q	0.0002381	0.0002381	0.000309
α	4.863	4.9174	5.499
β	14,957	15,936	11,193
β_γ	14,240	15,102	10,694
ϵ	1,312	0	0
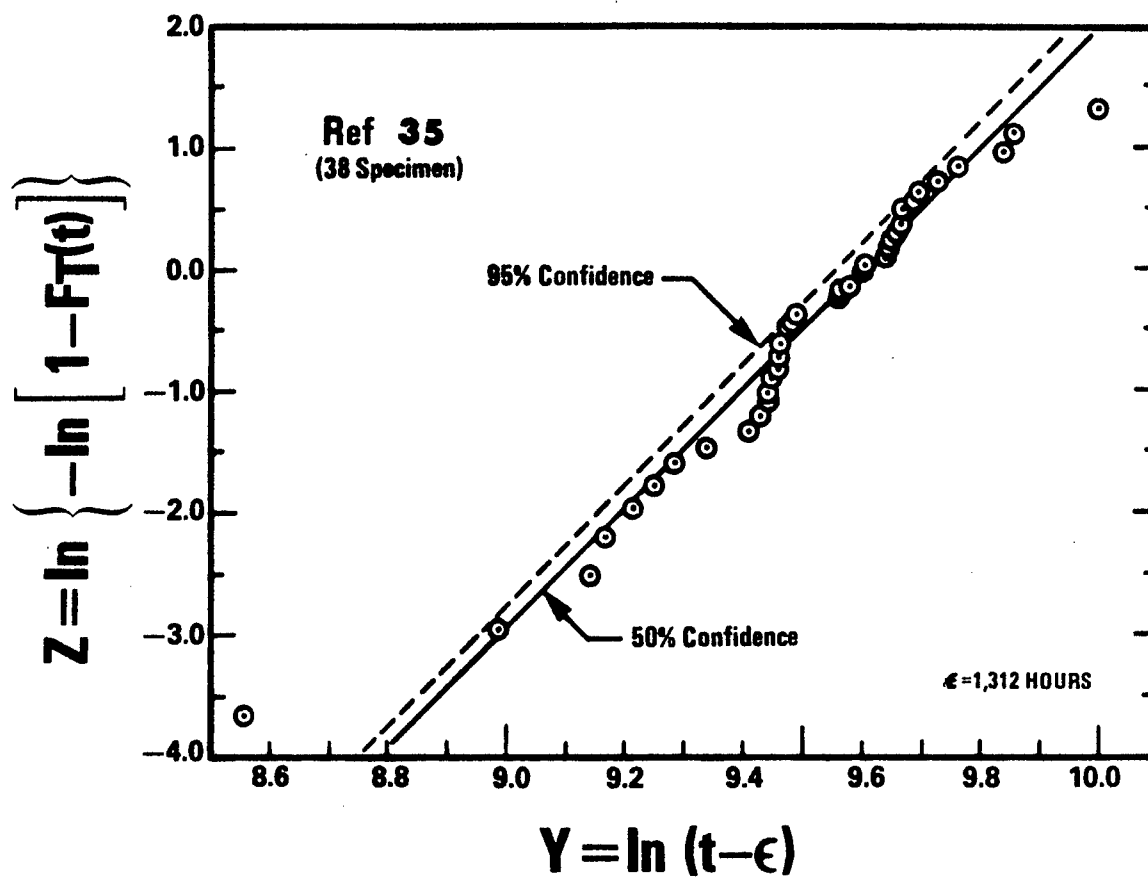
Figure 7 Weibull Best Fit Plot for WPF (38) Data Set
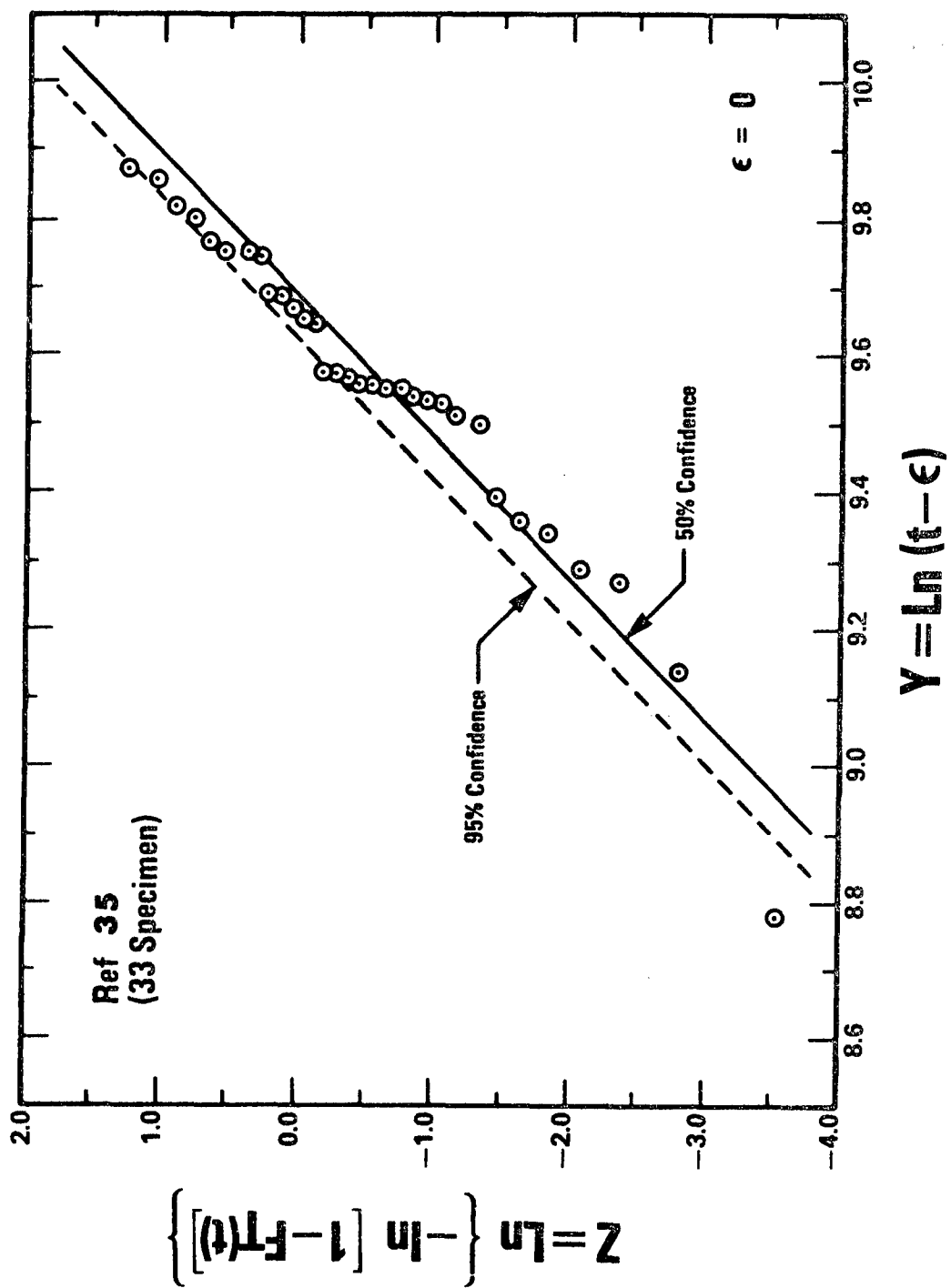
Figure 8 Weibull Best Fit Plot for WPF (33) Data Set
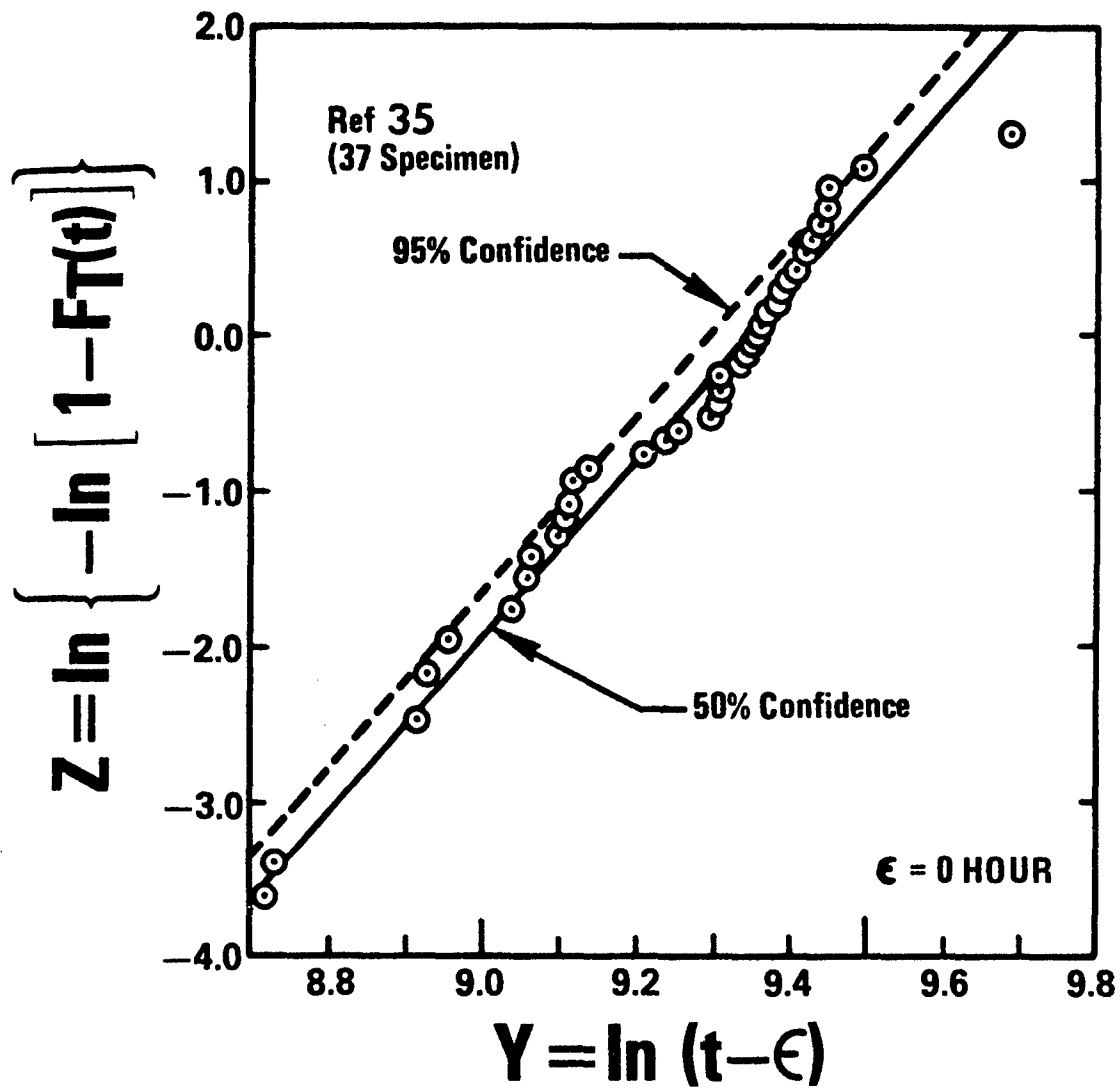
Figure 9 Weibull Best Fit Plot for XWPF Data Set
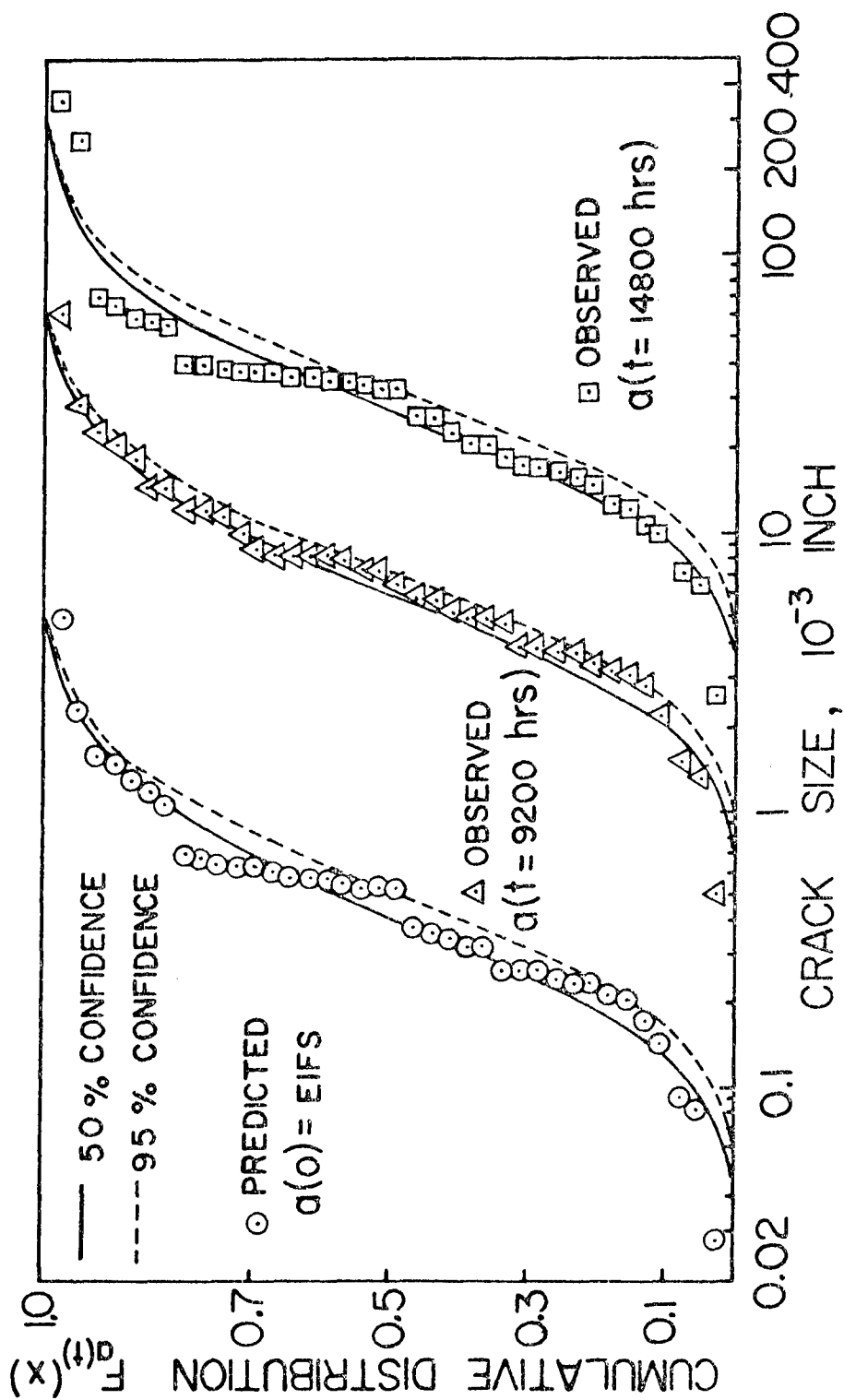
Figure 10 Crack Size Versus Cumulative Distribution
$F_{a(t)}(x)$ for WPF (38) Data Set
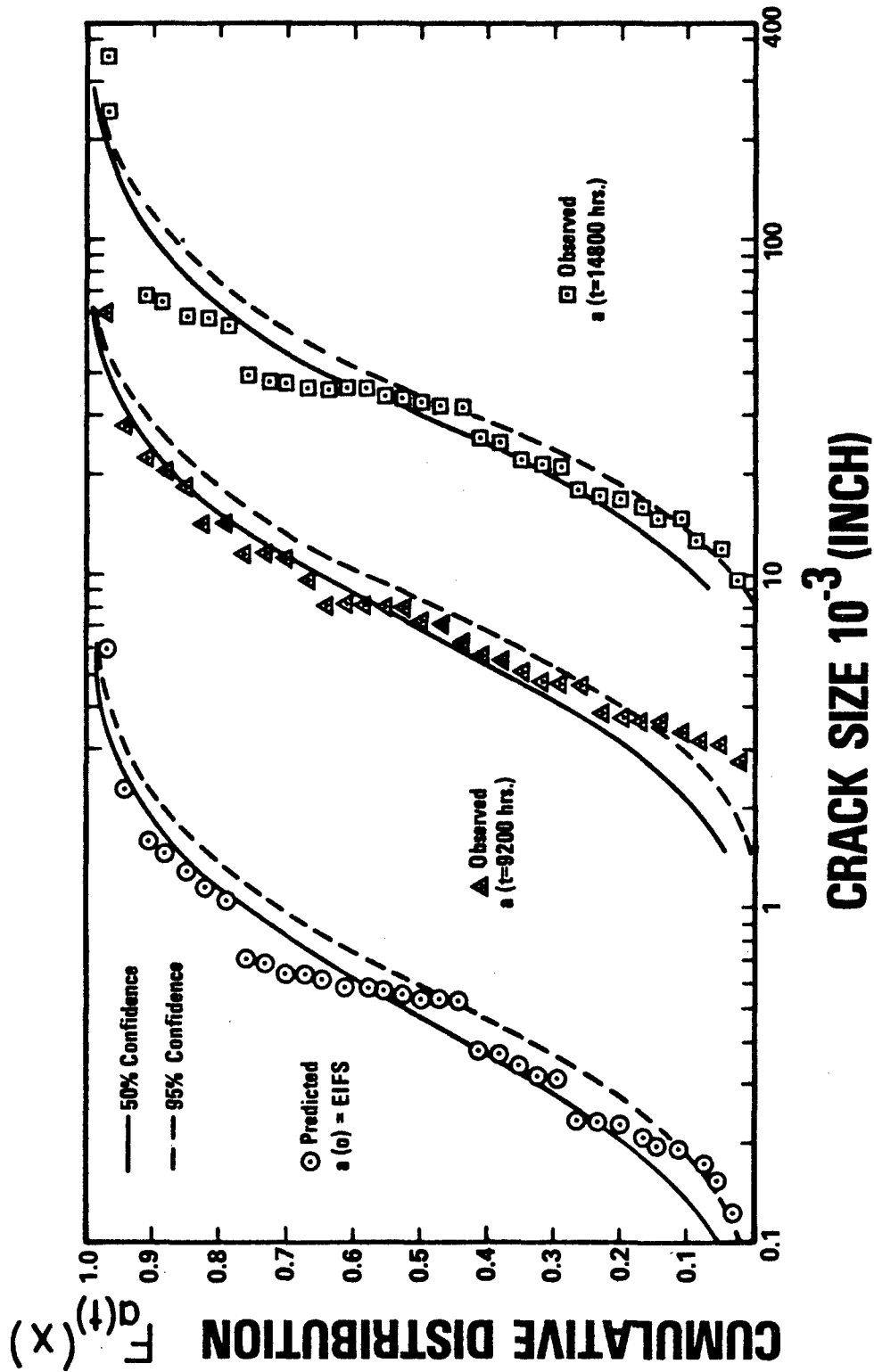
Figure 11 Crack Size Versus Cumulative Distribution $F_{a(t)}(x)$ for WPF (33) Data Set

29
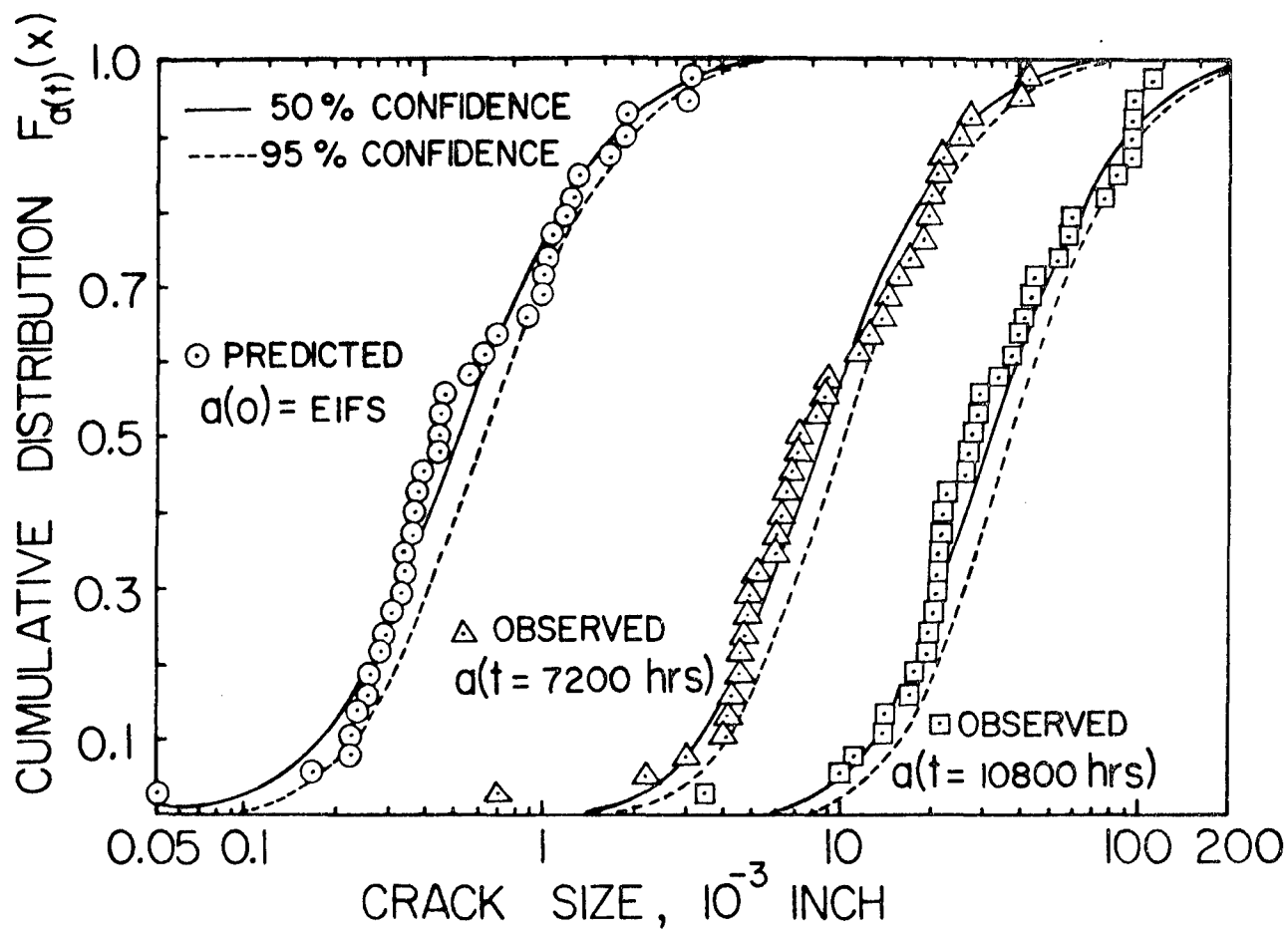
Figure 12   Crack Size Versus Cumulative Distribution $F_{a(t)}(x)$ for XWPF Data Set

50 % CONFIDENCE
95 % CONFIDENCE

⊙ PREDICTED
a(o) = EIFS

△ OBSERVED
a(t = 7200 hrs)

▢ OBSERVED
a(t = 10800 hrs)

CUMULATIVE DISTRIBUTION $F_{a(t)}(x)$

1.0

0.7

0.5

0.3

0.1

0.05 0.1   1   10   100 200

CRACK SIZE, 10^{-3} INCH

Other statistical distributions, such as the normal, lognormal, Weibull, Pearson, and Johnson,have been used to fit the test results [36]. The Kolmogorov-Smirnov test for goodness of fit indicates the presented theoretical distribution, Eq. (12) , referred to as the Weibull-compatible distribution, fits the test results best.

The predicted cumulative distribution of crack sizes $F_{a(t)}(x)$ tracks observed crack sizes very well (Figs. (10) through (12)). The EIFS distribution at t=0 is not only compatible with the observed TTCI results, but it is also statistically compatible with the observed crack size distributions. Although EIFS cracks are not observable, the above approach provides an indirect verification of the proposed EIFS distribution, Eq. (12).

3.6 Conclusions

A physically meaningful EIFS distribution has been suggested herein. The distribution is derived from the three-parameter Weibull distribution that is used for the distribution of time to crack initiation. Since the Weibull distribution satisfies the condition of the fatigue wear-out process [24-27], the EIFS distribution suggested herein bears the physical meaning of the fatigue wear-out process. It is shown that the correlation between the predicted crack size distribution at time t, extrapolated from the EIFS distribution, and the observed crack sizes at time t is excellent.

SECTION IV

CRACK GROWTH DAMAGE ACCUMULATION

After the statistical distribution of the equivalent initial flaw size is established from the test results of TTCI described previously, the entire population of the EIFS is subjected to crack propagation in service. Because of the crack geometry, the effect of loading sequence (e.g., retardation and acceleration), and many other factors, the crack growth rate equation given by Eq. (8) does not hold for the entire region of the crack size. As a result, crack growth predictions must be carried out numerically by use of a cycle-by-cycle integration approach using a general computer program. This program must account for all the variables influencing the crack growth behavior such as crack geometry, load sequence effect, and many others. The crack size $a(t_2)$ at t_2 flight hours can be expressed in terms of $a(t_1)$, where $t_1 \leq t_2$ as follows, [e.g., Ref. 6] :

$$a(t_2) = a(t_1) + \Sigma \, \Delta a(t_j) \tag{20}$$

in which $\Delta a(t_j)$ is the crack growth increment per flight hour at t_j where $t_1 \leq t_j \leq t_2$.

Thus, the crack growth damage $a(t)$ as a function of the service time t is determined from the general computer program starting from a crack size much smaller than the EIFS. Such an analytical crack growth curve is referred to as the "master curve".

Two master curves for the fastener holes of 7475-T7351 aluminum specimens with no load transfer and Winslow drilled with proper drilling techniques under a fighter spectrum are presented in Fig. (13). Curves 1 and 2 are associated with different maximum stress levels σ_{max} in the fighter spectrum as shown in the figure. Furthermore, Fig. (14) indicates two master curves for the same fastener hole conditions and the same maximum stress levels except with a 15% load transfer. Both Figs. (13) and (14) are established using the general crack growth damage accumulation computer code developed by General Dynamics.

The crack growth rates corresponding to Figs. (13) and (14) are presented in Figs. (15) and (16) respectively. It is observed from these figures that in the small crack size region the crack growth rate vs. the crack size in log scale is practically a straight line, indicating the validity of Eq. (8).
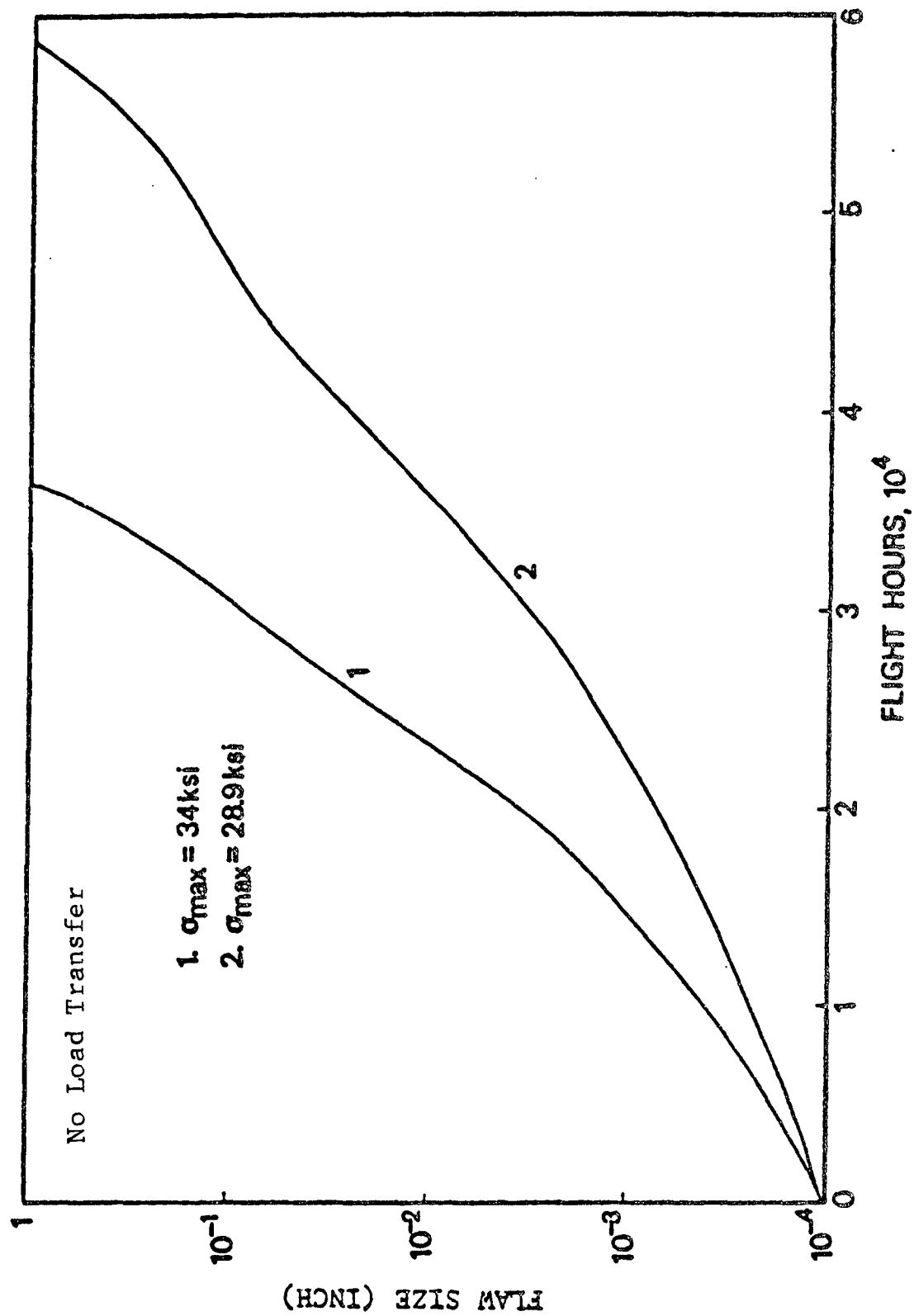
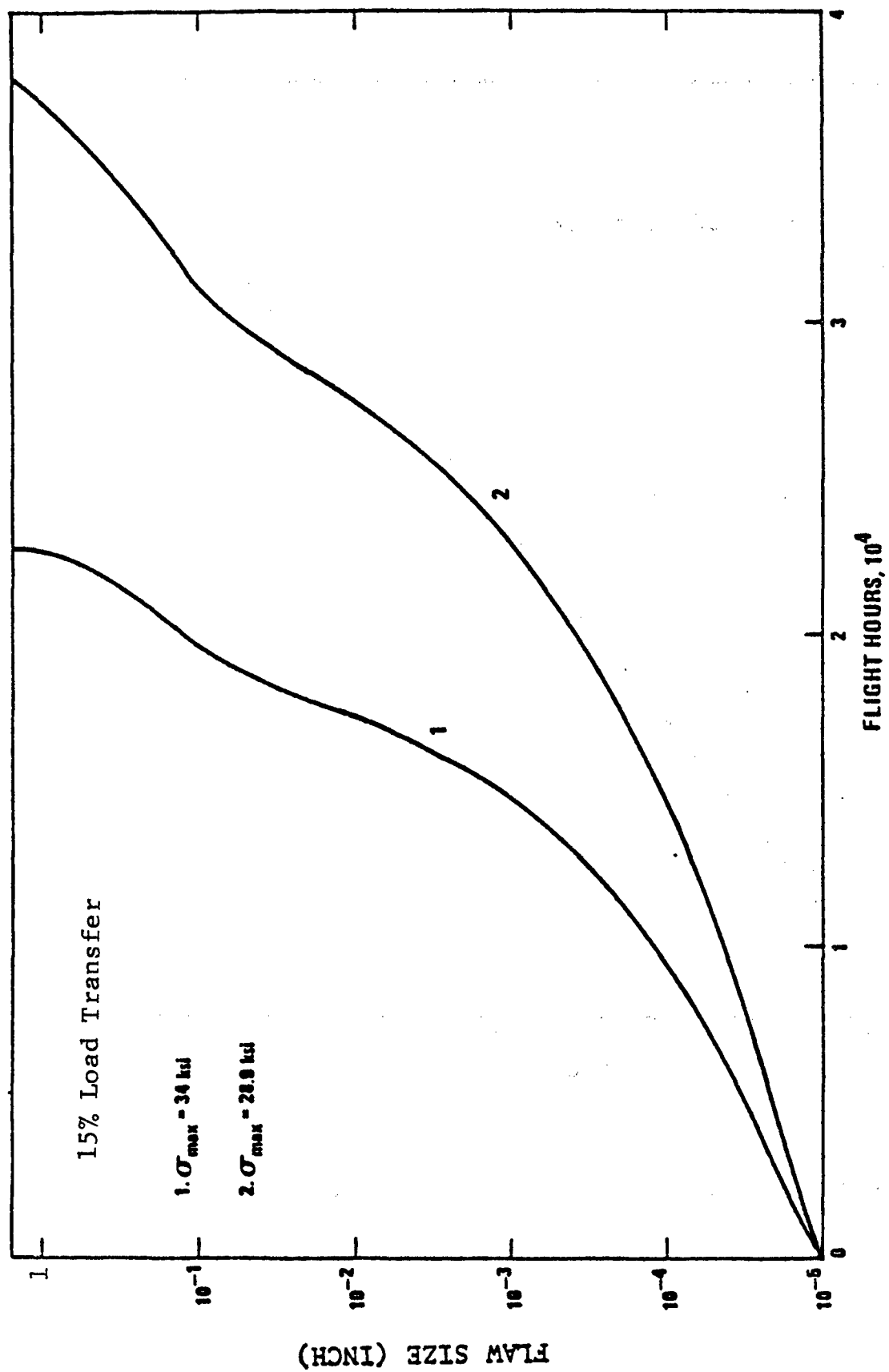Figure 13   Flaw Size Versus Flight Hours   (No Load Transfer)

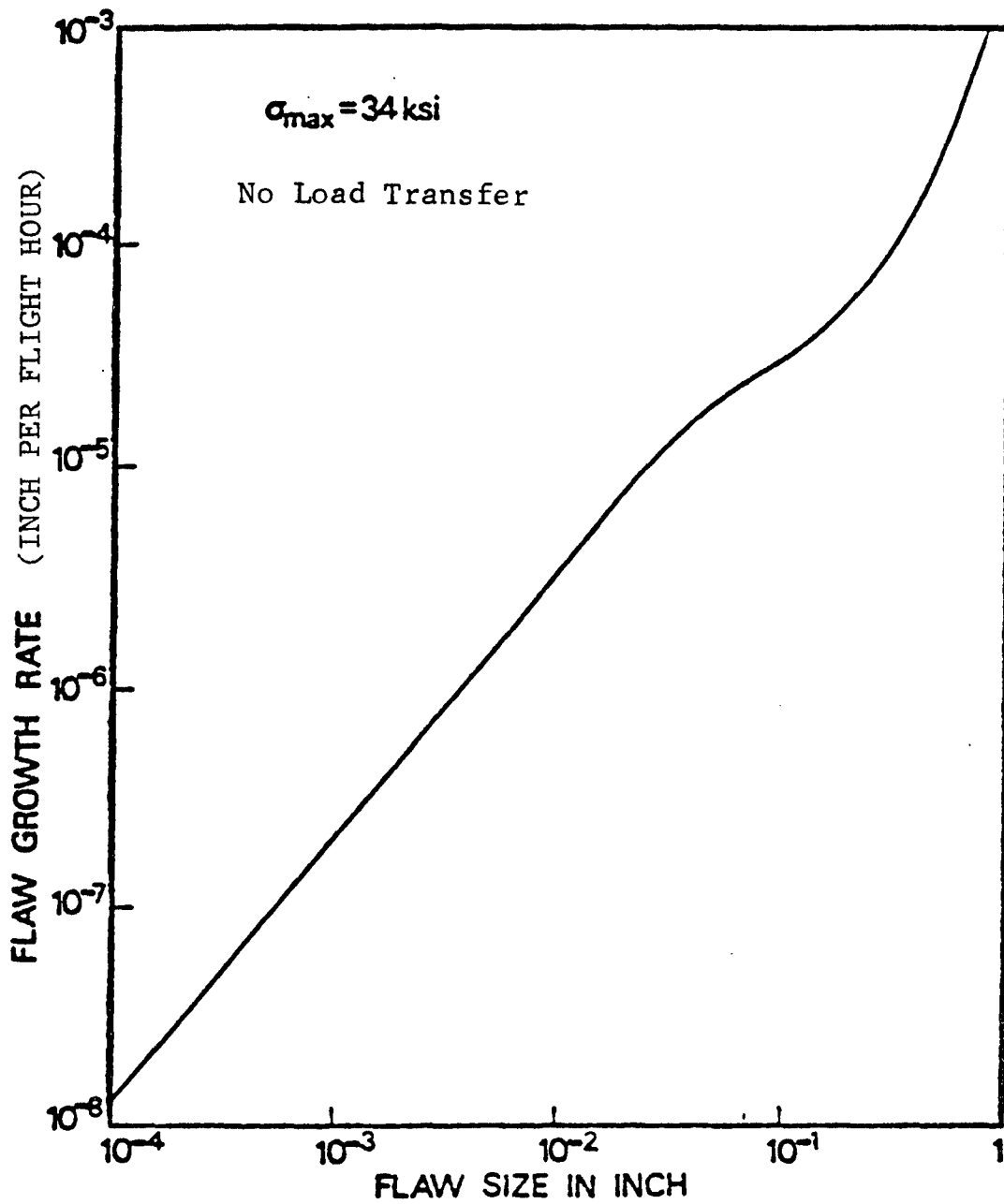Figure 14   Flaw Size Versus Flight Hours (15% Load Transfer)

35

Figure 15   Flaw Growth Rate Versus Flaw Size (No Load Transfer)
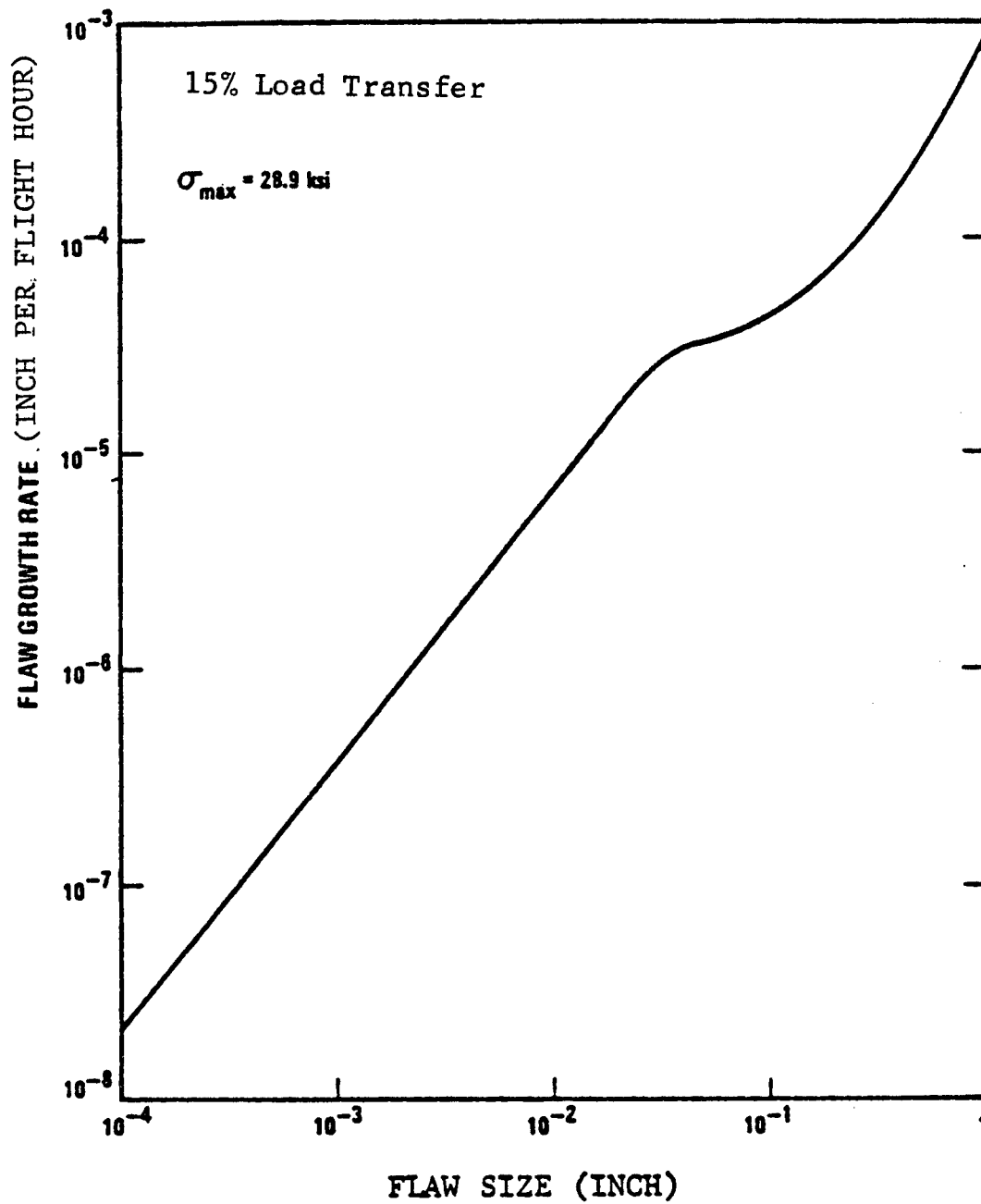
Figure 16 Flaw Growth Rate Versus Flaw Size (15% Load Transfer)

37

It follows from the particular case of Eq. 8 and Eqs. (A-1) and (A-2) from the Appendix that the relationship between the crack sizes $a(t_1)$ and $a(t_2)$ at t_1 and t_2 does not depend on t_1 and t_2 but only on t_2-t_1. This relationship is not valid for the general case within one flight of a particular mission, because of the crack size and the load sequence effect. Thus, within one flight of a particular mission, the relationship between $a(t_1)$ and $a(t_2)$ represented by Eq. (20) depends on both t_1 and t_2. Since, however, the design loading spectra in one lifetime consists of many repeated missions and flights, it is reasonable to assume that the relationship between $a(t_1)$ and $a(t_2)$ depends only on the time difference t_2-t_1 of the service time. Such an expedient approximation appears to be acceptable and it simplifies the computational procedures of the durability analysis tremendously. As a result, only a crack growth master curve for each maximum stress level in the loading spectra is sufficient for the estimation of the crack growth damage accumulation in the durability analysis.

Thus for the purpose of mathematical derivation, the analytical master curve $a(t)$ in the ith stress region can be symbolically represented by the following equation in terms of t_2-t_1

$$a(t_1) = W[a(t_2), \ t_2-t_1] \tag{21}$$

in which W is a general function representing the master curve. Eq. (21) indicates that for a given value of crack size $a(t_2)$ at t_2, one can determine the crack size $a(t_1)$ at t_1 from the master curve as depicted in Fig. (17). Eq. (21) will be used in the mathematical development of the durability analysis for predicting the crack growth damage accumulation of the crack population. It will serve, mathematically, as the transfer function for transferring the statistical distributions of the crack population from one service time to another, as will be described later.

It should be emphasized that in the present analysis, any general crack growth damage accumulation package and method [6,7,37-47] can be used to obtain the master curve. It appears that the miniblock approach proposed by Gallagher [37-39] using the flight-by-flight integration technique is most efficient for the present purpose. Furthermore, the master curve varies from one stress region to another in the entire durability critical component. Hence, an appropriate master curve should be used for a particular stress region.
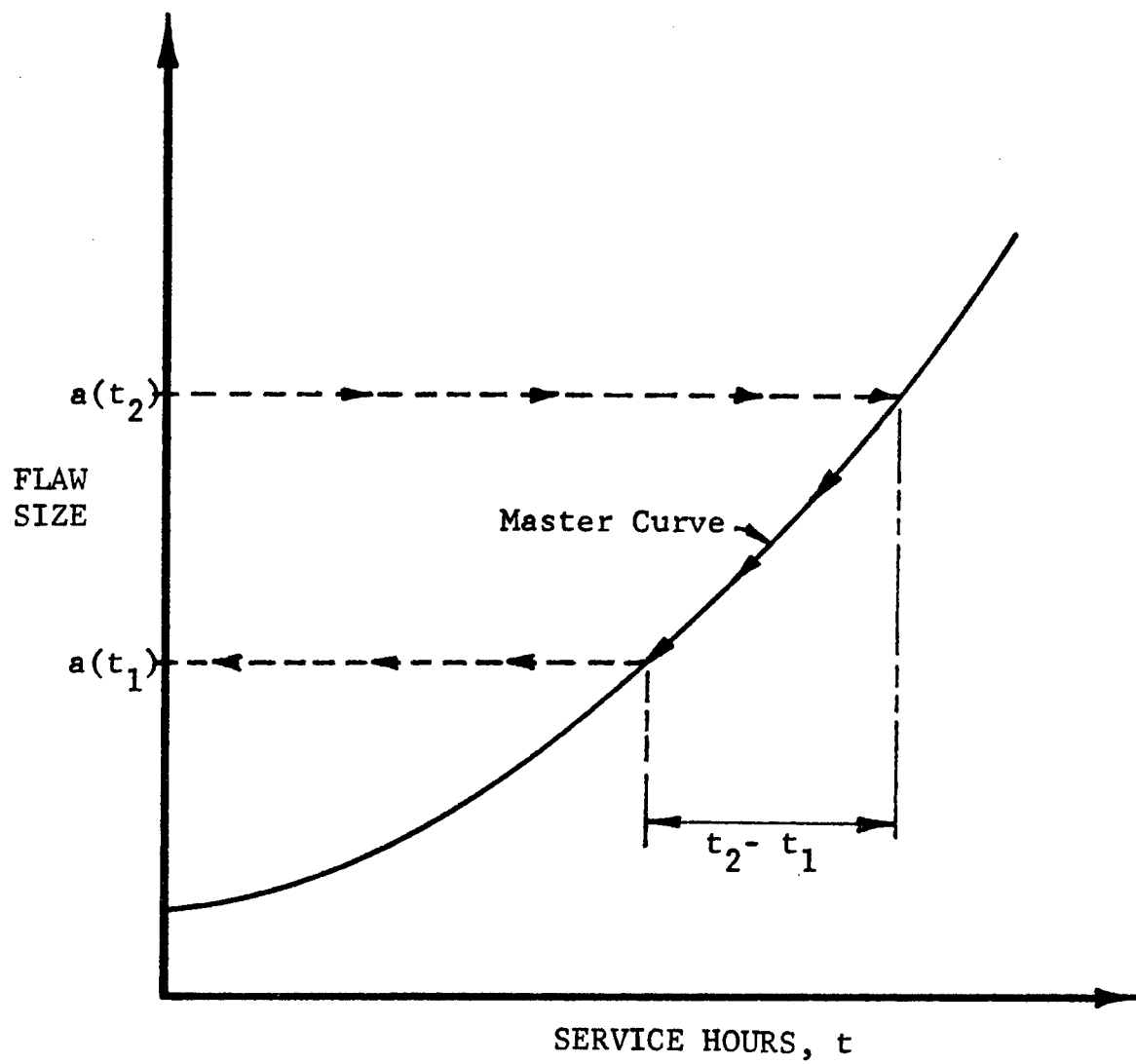
Figure 17   Interpretation of W Function

SECTION V

CRACK EXCEEDANCE AND MAINTENANCE COST

A durability critical component is divided into m stress regions. The maximum stress level in each stress region at every location or detail (e.g., fastener hole) is approximately the same, whereas the maximum stress level varies from one stress region to another. A wing panel of a F-16 fighter aircraft, which is identified as a durability critical component, is shown in Fig. (18). The lower skin of the wing panel is divided into three stress regions where the maximum applied stresses (limit) to each region under the fighter spectrum are 24.3 ksi, 27.0 and 28.3 as shown in Fig. (19).

Let N_i be the total number of details in the ith stress region. For instance, the total number of fastener holes in each region of Fig. (19) is, respectively, 59, 335 and 1220. If the maximum stress level varies from one location to another, then $N_i=1$.

In the ith stress region, let $N(i,\tau)$ represent the total number of details having a crack size exceeding x_1 at any service time τ. It is obvious that $N(i,\tau)$ is a statistical (random) variable, since the initial flaw size is a statistical variable. It is mentioned that each detail such as the fastener hole may have multiple cracks. In this report, the crack of a detail refers to the largest crack, and hence, each detail has one crack (the largest one). As a result, the term "crack" is interchangeable with "detail".

It is reasonable to assume that the crack growth at each detail is not influenced by the cracks in its neighboring details and that the crack damage accumulation of each detail is statistically independent. Then, the statistical distribution of $N(i,\tau)$ can be shown to follow the Binomial distribution

$$P\left[N(i,\tau) = n\right] = \binom{N_i}{n} p^n(i,\tau)\left[1-p(i,\tau)\right]^{N_i-n} \qquad (22)$$

in which $P[N(i,\tau) = n]$ denotes the probability that the total number of details having a crack size exceeding x_1 in the ith stress region, $N(i,\tau)$, is equal to n, and $p(i,\tau)$ is the probability that a detail in the ith stress region will have a crack size greater than x_1 at the service time τ. In Eq. (22) , $\binom{N_i}{n}$ is the combination of n out of N_i, i.e.,

$$\binom{N_i}{n} = \frac{N_i!}{n!(N_i-n)!} = \frac{\Gamma(N_i+1)}{\Gamma(n+1)\,\Gamma(N_i-n+1)} \qquad (23)$$

41

Upper Attach Fittings

16W101 Upper Wing Skin

16W138 No. 8 Spar

16W137 No. 7 Spar

16W136 No. 6 Spar

16W139 No. 9 Spar

16W135 No. 5 Spar

16W134 No. 4 Spar

16W102 Lower Wing Skin

Lower Attach Fittings
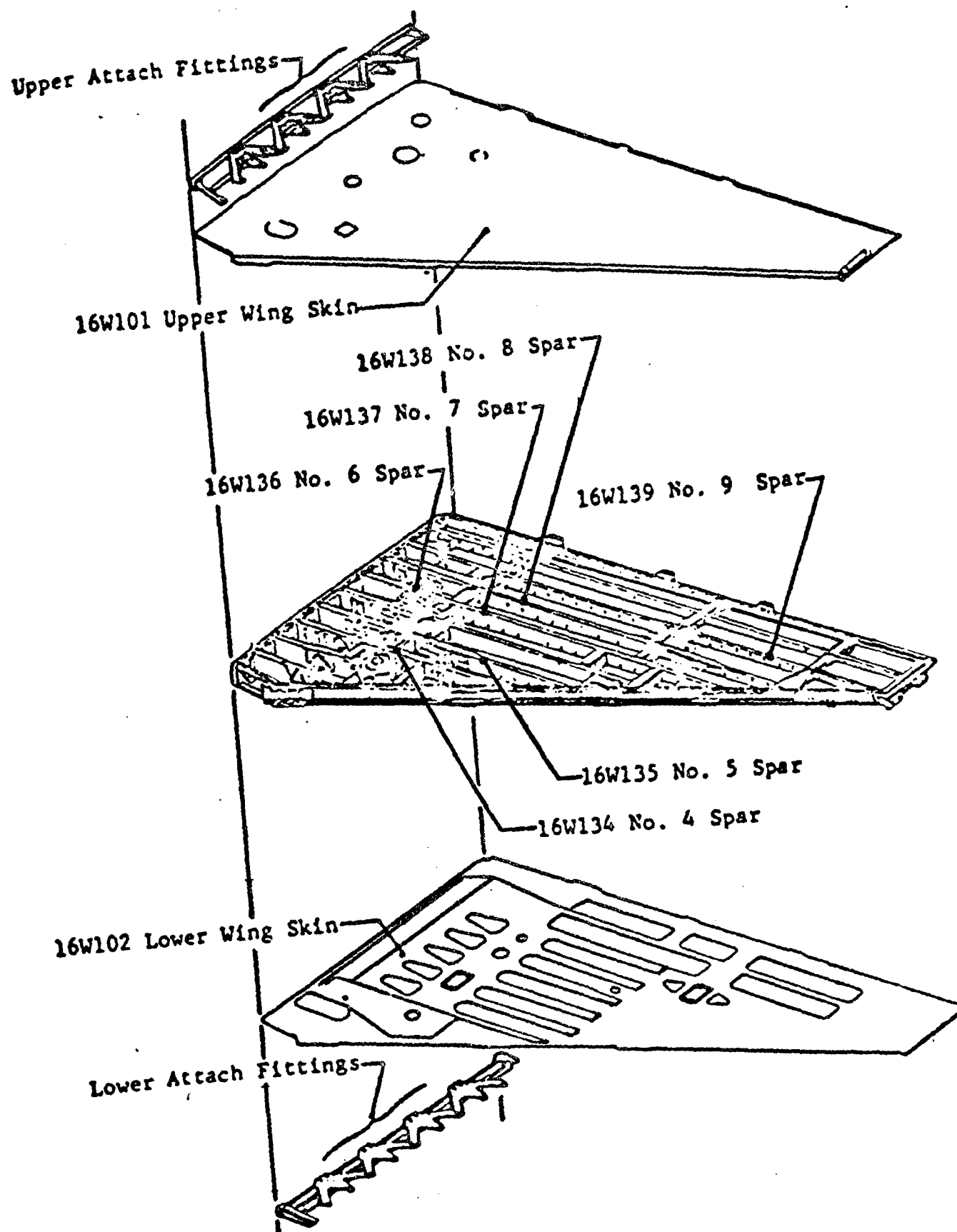
Figure 18 F-16 Basic Wing Structure

LEGEND: APPROXIMATE NUMBER OF FASTENERS PER STRESS RANGE

27.0-28.3 ksi Limit (59 Holes)

24.3-27.0 ksi Limit (335 Holes)

Less Than 21.6-24.3 ksi Limit (1220 Holes)
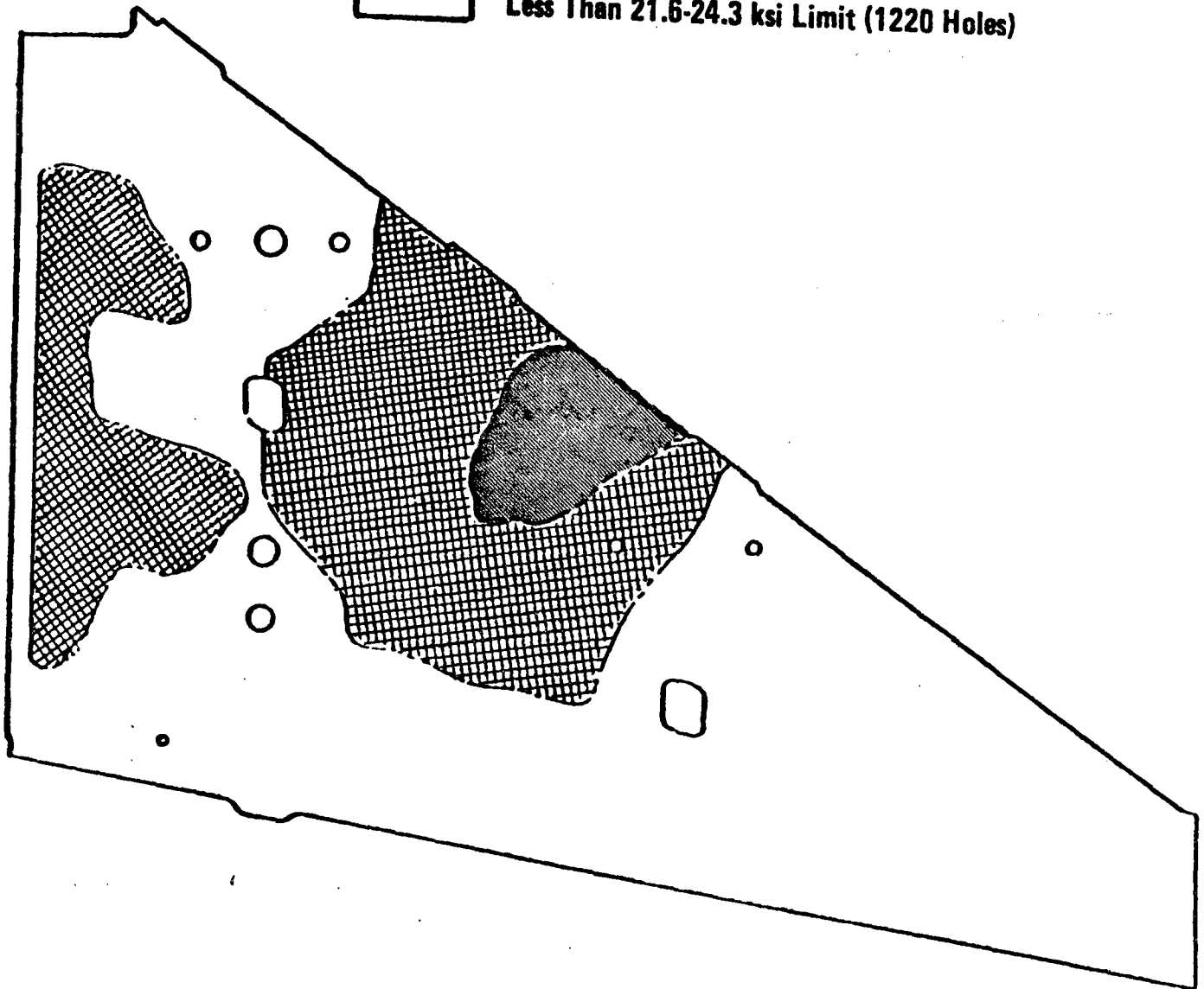
Figure 19   Stress Zoning for Durability Component Wing-Lower Skin

in which $\Gamma(.)$ is the gamma function.

The average value, $\bar{N}(i,\tau)$, and the variance, $\sigma_N^2(i,\tau)$, of $N(i,\tau)$ (the total number of details in ith stress region having a crack size greater than x_1 at any service time τ) can be obtained from Eq. (22) as follows

$$\bar{N}(i,\tau) = \sum_{n=0}^{\infty} nP[N(i,\tau) = n] = N_i p(i,\tau) \qquad (24)$$

$$\sigma_N^2(i,\tau) = \sum_{n=0}^{\infty} n^2 P[N(i,\tau) = n] - \bar{N}^2(i,\tau) \Bigg\} $$
$$= N_i p(i,\tau) \Big[1 - p(i,\tau)\Big] \qquad (25)$$

If $N*$ denotes the total number of details in the entire durability critical component and $L(\tau)$ indicates the total number of details in the entire component having a crack size greater than x_1 at the service time τ, then it is obvious that

$$N* = \sum_{i=1}^{m} N_i \qquad (26)$$

$$L(\tau) = \sum_{i=1}^{m} N(i,\tau) \qquad (27)$$

Thus, the average value, $\bar{L}(\tau)$, and the variance $\sigma_L^2(\tau)$, of $L(\tau)$ are obtained, respectively,

$$\bar{L}(\tau) = \sum_{i=1}^{m} \bar{N}(i,\tau) \qquad (28)$$

$$\sigma_L^2(\tau) = \sum_{i=1}^{m} \sigma_N^2(i,\tau) \qquad (29)$$

in which $\bar{N}(i,\tau)$ and $\sigma_N^2(i,\tau)$ are given by Eqs. (24) and (25) .

The total number of details in the entire component, $L(\tau)$, having a crack size larger than x_1 at the service time τ is a statistical variable. It follows from Eq. (27) that $L(\tau)$ is the sum of independent Binomial variables, Eq. (22).

When the number of details in each stress region, N_i, is large, the Binomial distribution given by Eq. (22) can be approximated by the Normal distribution, in which case, the statistical distribution of $L(\tau)$ is also Normal with the mean and the variance given by Eqs. (28) and (29). The approximation by the Normal distribution does simplify the computational effort.

The component may undergo a scheduled inspection and repair maintenance procedure at T_1, T_2, T_3 ... in service with service intervals τ_1, τ_2, τ_3, ... as shown in Figure (1). It is assumed that the cracked details detected during inspection will be repaired. Let $g_j(x)dx$ be the probability of detecting (or repairing) a crack in the size range $(x, x + dx)$ at the jth inspection. Then the average maintenance cost, consisting of the cost of inspections and repairs, in the service interval $(0, T_{I+1})$, denoted by $\overline{C}(I)$, is

$$\overline{C}(I) = C_1 N^* I + \sum_{j=1}^{I} \int_0^\infty C_2(x) g_j(x) dx \qquad (30)$$

in which C_1 = cost of inspecting one detail, I = total numbers of scheduled inspections, $C_2(x)$ = cost of repairing one cracked detail having a crack size x, and N^* is the total number of details in the entire component, Eq. (26).

The cost of repairing a crack, $C_2(x)$, depends on the crack size x. For the crack size smaller than 0.03", it is only necessary to ream the fastener hole to the next hole size and hence the cost of repair, $C_2(x)$, is identical for all $x \leq 0.03"$. For the crack size greater than 0.03", a regular retrofit procedure may be needed. For even larger cracks, a patching replacement of the entire component may be necessary which is much more expensive. Therefore, the cost of repairing a crack size x, $C_2(x)$, is in general an increasing function of the crack size x Likewise $C_2(x)$ may also be a discontinuous function of x as described above [Ref. 10].

The probability of detecting a crack size $g_j(x)dx$ in the size range $(x, x+dx)$ depends on the resolution capability of a particular NDI technique used at the jth inspection as well as the statistical distribution of the crack size at T_j. It is observed from Eqs. (22)-(30) that the estimation of the crack exceedance and the maintenance cost depends on $p(i, \tau)$ and $g_j(x)$. These two quantities, along with the structural reliability, will be derived in the next section.

45

SECTION VI

DERIVATION FOR $p(i,\tau)$ AND $g_j(x)$

Even under well-controlled laboratory conditions, the detection of a given crack size is a statistical variable and it should be specified statistically. Let $F_D(x)$ be the probability of detecting a crack size x and $F_D^*(x)=1-F_D(x)$ be the probability of missing it. Clearly, both $F_D(x)$ and $F_D^*(x)$ depend on the crack size x and the resolution capability of a particular NDE procudure [e.g., Refs. 48,49]. Various functional forms for $F_D(x)$ have been suggested [11,16].

6.1 WITHOUT INSPECTION AND REPAIR MAINTENANCE

Without scheduled inspection and repair maintenance, the probability $p(i,\tau)$ that the crack size $a(\tau)$ of a detail in the ith stress region will exceed a value x_1 at the service time τ is given by

$$p(i,\tau) = P\left[a(\tau) \geq x_1\right] = 1 - F_{a(\tau)}(x_1) \tag{31}$$

Since $a(\tau)$ is related to the initial crack size $a(0)$ through Eq. (21) in which $t_1=0$, $t_2=\tau$, i.e.,

$$a(0) = W[a(\tau),\tau] \tag{32}$$

Eq. (31) becomes

$$p(i,\tau) = P\left[a(0) \geq W(x_1,\tau)\right] = P\left[a(0) \geq y_1(\tau)\right]$$

or

$$p(i,\tau) = 1 - F_{a(0)}[y_1(\tau)] \tag{33}$$

in which $F_{a(0)}(x)$ is the distribution function of the equivalent initial flaw size given by Eq. (12) and argument $y_1(\tau)$ is related to x_1 as

$$y_1(\tau) = W(x_1,\tau) \tag{34}$$

The value of $y_1(\tau)$ given by Eq. (34) is obtained from the master curve for a given value of x_1 as shown in Fig. (20).

Thus $p(i,\tau)$ is obtained by substituting Eq. (12) into Eq. (33) as follows:

$$p(i,\tau) = 1 - \exp\left\{-\left[\frac{y_1^{-c}(\tau) - a_o^{-c} - cQ\varepsilon}{cQ\beta}\right]^{\alpha}\right\} \quad ; \quad y_1(\tau) \leq x_u \tag{35}$$

$$= 0 \quad ; \quad y_1(\tau) \geq x_u$$

in which x_u is given by Eq. (13). Note that $p(i,\tau)$ is a function of the crack size x_1 when Eq. (34) is substituted into Eq. (35).

For a special case in which the crack growth rate in the ith stress region follows Eq. (8), i.e., $da(t)/dt = Q_i a^{b_i}(t)$, the master curve can be written, after integration, as follows

$$a(0) = \frac{a(t)}{[1 + a^{c_i}(t)c_iQ_it]^{1/c_i}} = W[a(t),t] \tag{36}$$

where $c_i = b_i - 1$. Hence $y_1(\tau)$ appearing in Eq. (35) can be expressed using Eq. (36) in the following:

$$y_1(\tau) = \frac{x_1}{[1 + x_1^{c_i}c_iQ_i\tau]^{1/c_i}} = W[x_1,\tau] \tag{37}$$

6.2 SCHEDULED INSPECTION AND REPAIR MAINTENANCE

When the durability critical component is subjected to a scheduled inspection and repair maintenance at T_1, T_2, T_3, ...with service intervals τ_1, τ_2, τ_3, ...as shown in Fig. (1), the solution for $p(i,\tau)$ is more involved. In the first service interval, the entire population of the initial flaws is propagated following the master curve as shown in Fig. (2). During the first inspection and repair performed at $t=T_1$, some of the cracked details will be detected and repaired. The crack size of the repaired details, referred to as the renewal details, is assumed to have the same distribution as that of the EIFS given by Eq. (12). In other words, the possible cold-work effect and others are neglected. Thus, after each inspection and repair maintenance, a new population (renewal details) is introduced
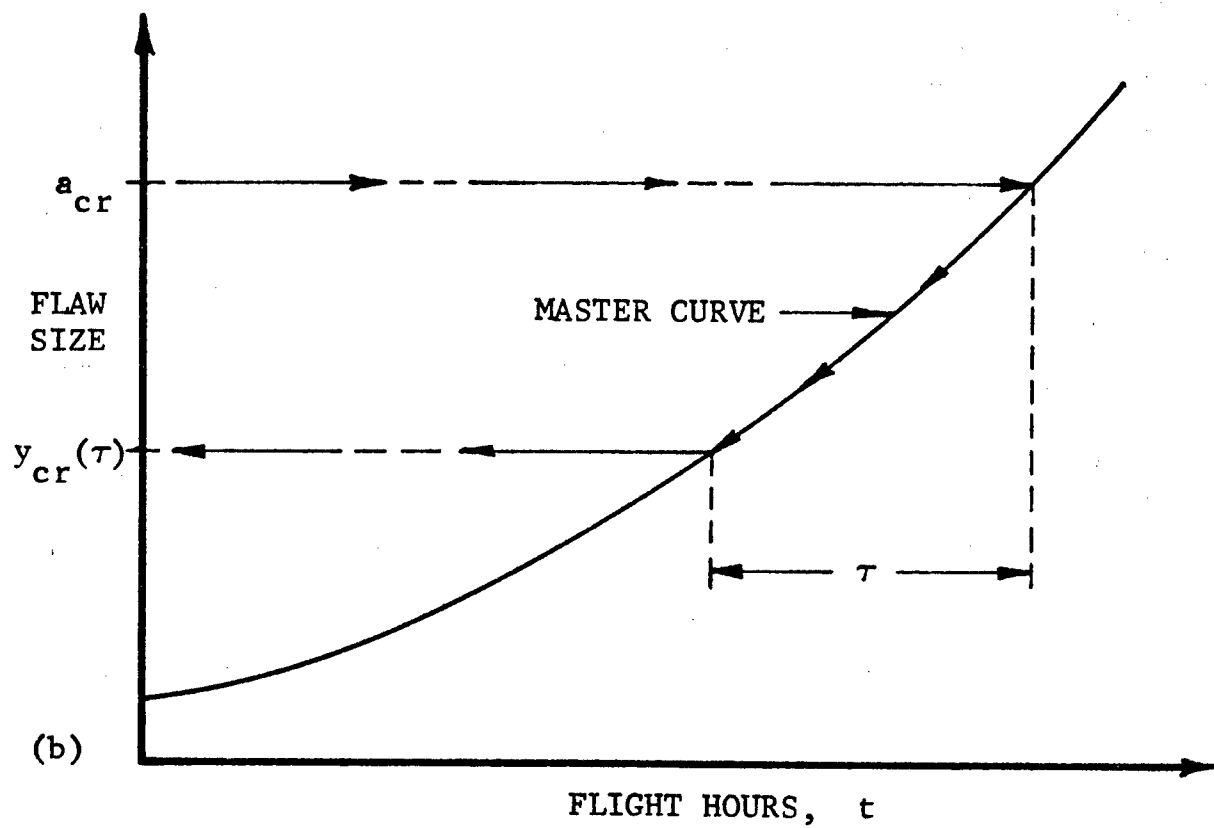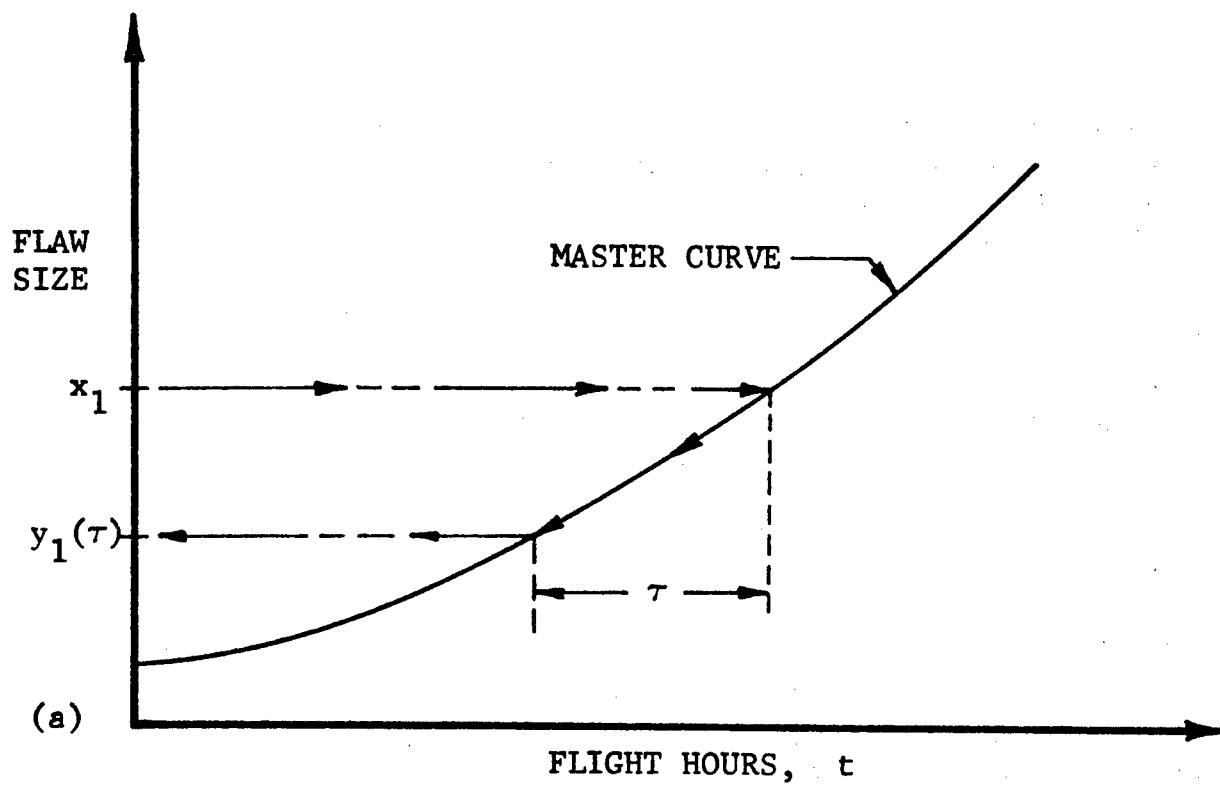
48

Figure 20 Computation of $y_1(\tau)$ and $y_{cr}(\tau)$

49

with certain probability depending on the crack size distribution prior to inspection as well as the detection probability $F_D(x)$ of a particular NDI procedure. Furthermore, the statistical distribution of the crack size of those undetected details is modified and truncated by the $F_D^*(x)$, i.e., the probability of nondetection.

Consequently, the distribution of the crack size of the entire population is continuously shifted and changed (to the large value) in service due to fatigue crack propagation. The distribution is also subjected to truncation and modification after each inspection and repair maintenance. Likewise, since an additional new population is introduced after each inspection and repair maintenance, the statistical distribution of the crack size after the jth inspection and repair maintenance will consist of j+1 different populations as shown in Fig. (21).

Referring to Figure (21), $f_{a(T_j)}(x)$ is the probability density of all cracks at T_j just before the jth inspection and repair (see Fig. (1)). $F_D(x)$ is the probability of being able to detect (and repair) cracks of size x. When these two are multiplied, the product represents the density of all cracks which are found and repaired. The area, $G(j)$, under the curve represents the percentage of repaired holes that will have a probability density of cracks corresponding to that of the EIFS, denoted by $f_{a(0)}(x)$. Cracks which are missed stay at the same length; that is, they will not be repaired. This portion is shown as $f_{a(T)}(x)$ then multiplied by $F_D^*(x)$ where $F_D^*(x)=1-F_D(x)$ is the probability of missing a crack size x. After repair, the probability density of cracks, $f_{a(T_j+)}(x)$, is composed of the sum of the undetected cracks, $f_{a(T_j)}(x)\, F_D^*(x)$, and repaired cracks $f_{a(0)}(x)G(j)$.

The inspection and repair procedure can be input into the analysis at any time desired. Thus, the effect of different inspection or repair procedures or intervals can be ascertained.

As a result, a new population is introduced after each inspection and repair maintenance. The variation of the probability density of the flaw size in service due to crack propagation and maintenance procedures is displayed in Fig. (21).

Let $f_{a(0)}(x)$ be the probability density function of the EIFS, $a(0)$. Since the probability density function is the derivative of the distribution function, one has

$$f_{a(0)}(x) = \frac{dF_{a(0)}(x)}{dx} \tag{38}$$

in which the distribution function $F_{a(0)}(x)$ of $a(0)$ is given by Eq. (12).

Top-left box:
$f_{a(T_j)}(x)F_D(x)$
Area= G(j) fraction of repaired cracks
CRACK SIZE a
POPULATION OF REPAIRED CRACKS

Top-right box:
$f_{a(T_j^*)}(x)$
CRACK SIZE a
POPULATION OF CRACKS AFTER REPAIR

=

=

Middle-left box:
$F_D(x)$
Detection Limits
1.0
CRACK SIZE a
CRACK DETECTION PROBABILITY

Middle-right box:
$f_{a(T_j)}F_D^*(x)$
CRACK SIZE a
MISSED CRACK POPULATION

×

+

Bottom-left box:
$f_{a(T_j)}(x)$
CRACK SIZE a
CRACK POPULATION

Bottom-right box:
$f_{a(0)}(x)G(j)$
CRACK SIZE a
REPAIRED CRACK POPULATION

Figure 21   Effect of Repair on Crack Population

51

Substitution of Eq. (12) into Eq. (38) leads to the expression for the probability density function of the equivalent initial flaw size

$$
f_{a(0)}(x) = \frac{\alpha x^{-c-1}}{Q\beta} [\phi(x)]^{\alpha-1} \exp\left\{-[\phi(x)]^{\alpha}\right\}
$$

$$
\phi(x) = \frac{x^{-c} - a_0^{-c} - cQ\epsilon}{cQ\beta}
$$

(39)

The probability $p(i,\tau)$ that the crack size of a detail in the ith stress region at any service time $\tau = T_j + t$ will exceed a value x_1, see Fig. (1), for $j = 0,1,2,\ldots$ is given by

$$
p(i,\tau) = \int_{x_1}^{\infty} f_{a(T_j+t)}(x)\,dx
$$

(40)

in which $f_{a(T_j+t)}(x)$ is the probability density function of the crack size $a(T_j+t)$, and t is any value smaller than or equal to $\tau_{j+1} = T_{j+1} - T_j$ for $j = 0,1,2,\ldots$, i.e., $t \leq T_{j+1} - T_j$. Hence, $a(T_j+t)$ is the crack size at any service time $T_j + t$ in the j+1th service interval (see Fig. (1)).

The probability density function, $f_{a(T_j+t)}(x)$, of the crack size $a(T_j+t)$, at any service time $\tau = T_j + t (j = 0,1,2,\ldots)$ is derived in the Appendix for both the special case and the general use. The results are given in the following;

6.2.1 In The First Service Interval, j=0

In the first service interval, $p(i,\tau)$ is identical to that given by Eqs. (34) and (35). The transformation of the density functions with the relationship $a(0) = W[a(t),t]$ can be found

$$
f_{a(t)}(x) = A_1(t;j) = f_{a(0)}[y_1(t)]I_1(t)
$$

(41)

52

in which

$$y_1(t) = W(x,t)$$

$$I_1(t) = \frac{dW(x,t)}{dx} = \left[\frac{dy_1(t)}{dx}\right] \Big/ \left[\frac{dx}{dt}\right] \qquad (42)$$

where the crack size, $y_1(t)$, and the corresponding crack growth rate, $dy_1(t)/dt$ (slope), are obtained from the crack growth master curve as shown in Fig. (22 (a)). The crack growth rate, dx/dt, when the crack size is x, is also obtained from the master curve, Fig.(22 (a)).

6.2.2 After the First Service Interval, $j=1,2,\ldots$

$$f_{a(T_j+t)}(x) = A_{j+1}(t;j) + \sum_{k=1}^{j} G(j-k+1) A_k(t;j) \qquad (43)$$

$$A_k(t;j) = \left\{ \prod_{m=1}^{k-1} F_D^*\left[y_m(t;j)\right] \right\} f_{a(0)}\left[y_k(t;j)\right] I_k(t;j) \qquad (44)$$

$$k=2,3,\ldots,j+1$$

$$y_m(t;j) = y_1\left[t + \sum_{n=j-m+2}^{j} \tau_n\right] = W\left[x, \ t + \sum_{n=j-m+2}^{j} \tau_n\right]$$

$$I_m(t;j) = \left[\frac{dy_m(t;j)}{dx}\right] = \left[\frac{dy_m(t;j)}{dx}\right] \Big/ \left[\frac{dx}{dt}\right] \qquad (45)$$

in which τ_n is the nth service interval as shown in Fig. (1).

The crack size, $y_m(t;j)$, and the crack growth rates (slopes), $dy_m(t;j)/dt$ and dx/dt, appearing in Eqs. (43)-(45)   are determined from the crack growth master curve as shown in Fig. (22). The analytical expressions of these quantities for a special case are given in the Appendix. Furthermore, $G(j)$ $(j=1,2,\ldots)$ appearing in Eq. (43) represents the probability of detecting a crack of any size at the jth inspection,

$$G(j) = \int_0^{\infty} g_j(x)\,dx \qquad (46)$$

53
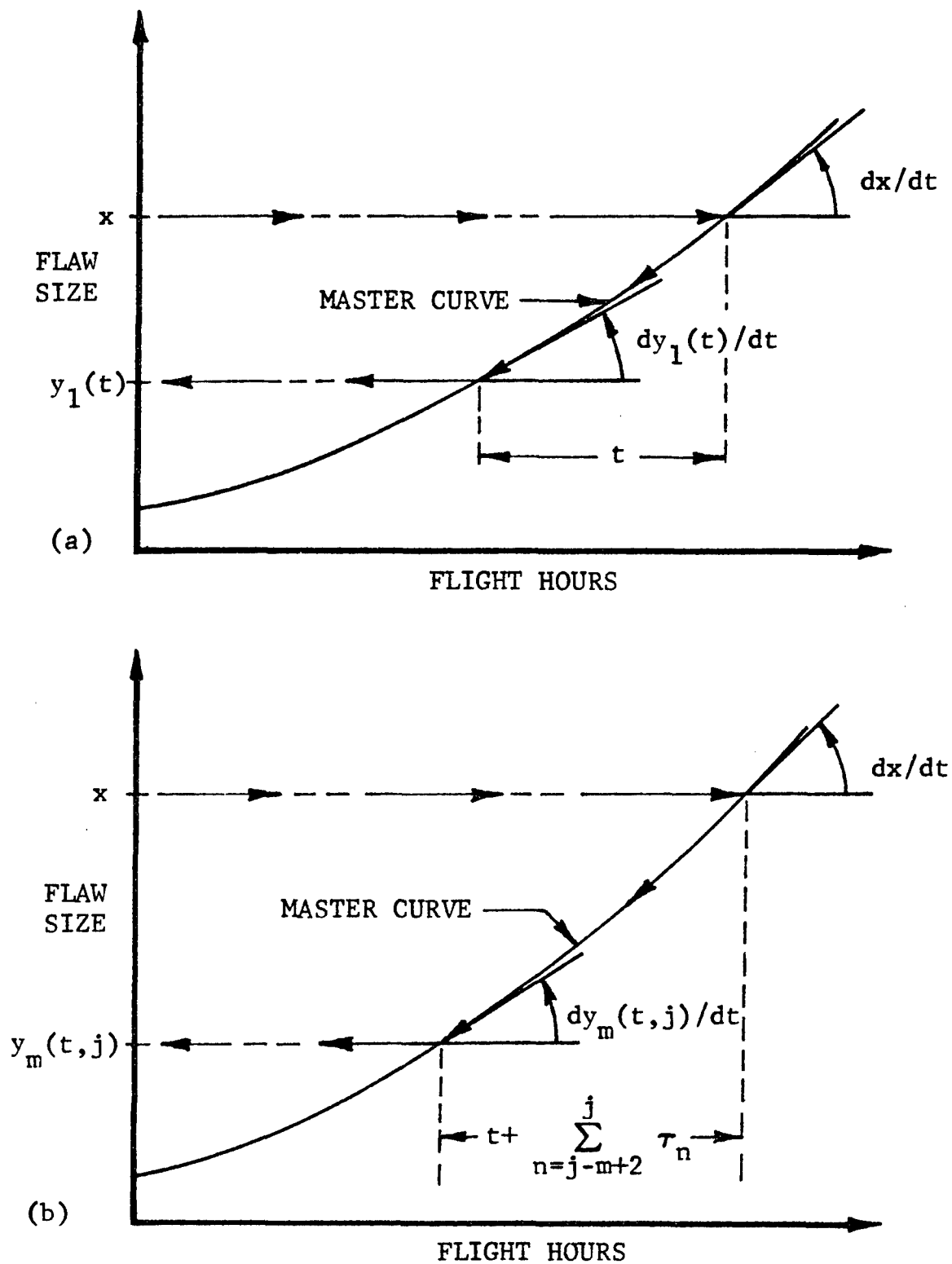
Figure 22   Utilization of Master Curves

in which $g_j(x)dx$ is the probability of detecting a crack in the size range $(x,x+dx)$ at the jth inspection T_j

$$g_j(x) = f_{a(T_j)}(x) F_D(x) \qquad (47)$$

Where $T_0 = 0$ and $f_{a(T_j)}(x) = f_{a(T_{j-1}+\tau_j)}(x)$ is the probability density function of the crack size $a(T_j)$ at T_j prior to the jth inspection, given by Eq. (43).

The probability of detecting (or repairing) a crack in the size range (z_1, z_2) at the jth inspection, i.e., T_j (j=1,2,...) flight hours, denoted by $q_j(z_1, z_2)$, follows from Eq. (47) as

$$q_j(z_1, z_2) = \int_{z_1}^{z_2} g_j(x)dx \qquad (48)$$

and the probability of detecting a crack size greater than z_1 at T_j, denoted by $q_j(z_1, \infty)$, is therefore

$$q_j(z_1, \infty) = \int_{z_1}^{\infty} g_j(x)dx \qquad (49)$$

which is referred to as the crack exceedance for the repaired cracks at the jth maintenance. It is obvious from Eq. (49) that

$$g_j(x) = -dq_j(x, \infty)/dx \qquad (50)$$

55

S T R U C T U R A L   R E L I A B I L I T Y ,
C R A C K   D E T E C T I O N   A N D   R E P A I R

The probability of detecting a crack in the size range (x,x+dx) at the jth inspection is given by Eq. (47) which depends on the detection probability $F_D(x)$ of a particular NDI technique. Various NDI procedures can be used for crack detection, such as visual inspection, liquid penetrant inspection, ultrasonic inspection, radiographic inspection, etc. The detection probabilities $F_D(x)$ under well-controlled laboratory conditions as a function of the crack size x have been available for various NDI techniques [e.g., Refs. 48-49]. The experimental results have been used as a basis to characterize the functional form of $F_D(x)$ for the purpose of engineering analyses [Refs. 11,14,16,17]. One functional form was proposed [Refs. 11,14].

$$
\begin{aligned}
F_D(x) &= 0 && \text{for } x \leq a_1 \\[2mm]
&= \left(\frac{x - a_1}{a_2 - a_1}\right)^m && \text{for } a_1 \leq x \leq a_2 \\[2mm]
&= 1 && \text{for } a_2 \leq x
\end{aligned}
\tag{51}
$$

in which a_1 is a crack size below which the crack cannot be detected and a_2 is a crack size beyond which the crack can be detected with certainty. Values a_1, a_2 and m should be determined to best fit the experimental results of a particular NDI procedure. Equation (51) has been used [e.g., Refs. 11,13,14,15] for engineering analyses.

Another functional form for the crack detection probability $F_D(x)$ was proposed in Ref. 16 as follows:

$$
\begin{aligned}
F_D(x) &= 0 && \text{for } x \leq a_1 \\[2mm]
&= c_1\left[1 - e^{-\beta_1(x-a_1)}\right] && \text{for } a_1 \leq x
\end{aligned}
\tag{52}
$$

in which a_1 has the same meaning as that in Eq. (51) and c_1 is a value slightly less than unity to allow for a small probability of missing a big crack during inspection. Again, the parameter values of a_1, c_1 and β_1 should be chosen to best fit the experimental results.

Sufficient information on the effectiveness of field inspection is not available for a characterization of $F_D(x)$. However, Eqs. (51) and (52) can be used with larger values of a_1, a_2, and c_1 should the field data become available. It is the general consensus that the effectiveness of the field inspection is much lower than that under laboratory conditions.

If a_{cr} denotes the critical crack size associated with the maximum design load P_{xx}, then the failure of a detail (such as a fastener hole) occurs when its crack size exceeds a_{cr}. Without a scheduled inspection and repair maintenance, the probability of failure of one detail in the ith stress region of the component within the service interval $(0,\tau)$, denoted by $P_i(0,\tau)$, follows from Eq. (31) as

$$P_i(0,\tau) = P[a(\tau) \geq a_{cr}] = 1 - F_{a(\tau)}(a_{cr})$$ (53)

Following the similar derivation for Eqs. (33) and (34) $P_i(0,\tau)$ can be shown as

$$P_i(0,\tau) = 1 - F_{a(0)}[y_{cr}(\tau)]$$ (54)

in which $F_{a(0)}(x)$ is the distribution function of the equivalent initial flaw size represented by Eq. (12), and $y_{cr}(\tau)$ is related to a_{cr} through Eq. (21) as

$$y_{cr}(\tau) = W(a_{cr},\tau)$$ (55)

For a given value of the critical crack size a_{cr}, $y_{cr}(\tau)$ is obtained from Eq. (55) using the crack growth master curve as shown in Fig. (20).

The probability of the survival of all the N_i details in the ith stress region of the component in the service interval $(0,\tau)$ is

$$\left[1 - P_i(0,\tau)\right]^{N_i}$$ (56)

58

Hence the probability of survival of the entire durability critical component consisting of m stress regions in the service interval $(0,\tau)$ denoted by $R(\tau)$, is

$$R(\tau) = \prod_{i=1}^{m} \left[1 - P_i(0,\tau)\right]^{N_i} \tag{57}$$

and the probability of failure of the entire component in $(0,\tau)$ is

$$P_f(\tau) = 1 - R(\tau) = 1 - \prod_{i=1}^{m} \left[1-P_i(0,\tau)\right]^{N_i} \tag{58}$$

Under the scheduled inspection and repair maintenance as shown in Fig. (1), the probability of failure of a detail in the ith stress region within the jth service interval (T_{j-1}, T_j) is given by

$$P_i(T_{j-1}, T_j) = \int_{a_{cr}}^{\infty} f_{a(T_j)}(x)\,dx , \quad \text{for } j=1,2,\ldots \tag{59}$$

in which $f_{a(T_j)}(x) = f_{a(T_{j-1}+\tau_j)}(x)$ is the probability density function of the crack size $a(T_j)$ prior to the jth inspection as derived in Eq. (43). The probability that a detail in the ith stress region will survive j service intervals $(0,T_j)$ is

$$\prod_{\ell=1}^{j} \left[1 - P_i(T_{\ell-1}, T_\ell)\right] \tag{60}$$

in which $T_0=0$. The probability that the ith stress region consisting of N_i details, will survive in the service interval $(0,T_j)$ is

$$\left\{ \prod_{\ell=1}^{j} \left[1 - P_i(T_{\ell-1}, T_\ell)\right] \right\}^{N_i} \tag{61}$$

59

Consequently, the probability of survival of the entire durability critical component which consists of m stress regions in the service interval $(0,T_j)$ is

$$R(T_j) = \prod_{i=1}^{m} \left\{ \prod_{\ell=1}^{j} \left[1 - P_i(T_{\ell-1}, T_\ell) \right] \right\}^{N_i} \tag{62}$$

and the probability of failure of the durability critical component in $(0,T_j)$ is

$$P_f(T_j) = 1 - \prod_{i=1}^{m} \left\{ \prod_{\ell=1}^{j} \left[1 - P_i(T_{\ell-1}, T_\ell) \right]^{N_i} \right\} \tag{63}$$

SECTION VIII

DEMONSTRATION OF METHODOLOGY

In order to demonstrate the applicability of the durability analysis methodology developed herein, the economic life of a hypothetical durability critical component of a fighter aircraft is considered for simplicity. A scheduled inspection and repair maintenance procedure is also considered and its effect on the economic life of the component is examined. The component consists of only one stress region having 100 identical fastener holes where the maximum stress level of the fighter spectrum for each fastener hole is σ_{max} = 34 ksi. Cracks are assumed to occur at the fastener holes only. The fastener holes of the component is assumed to be identical to those of the WPF data set presented in Table 1, page 22.

Test data of TTCI at 0.03 inches for WPF and XWPF data sets were presented in Tables 1 and 2, respectively, and summarized in Table 4. The three-parameter Weibull distribution, Eq. (2), was used to best fit the experimental data as shown in Figs. (8) and (9). The parameter values of α, β, ε and β_γ for γ=95% confidence level are presented in Tables 3 and 5. The crack growth master curves and the associated crack growth rates obtained using the CGR computer code for crack growth damage accumulation were presented in Figs. (13) to (16). The corresponding b and Q values in the small crack size range are given in Table 5.

With the aid of Eq. (9), the backward extrapolations have been carried out to obtain the corresponding equivalent initial flaw sizes from the test data of times to crack initiation. The results are presented in Table 4 and plotted in Figs. (23) and (24) as circles. Also plotted in Figs. (23) and (24) as a solid curve and a dashed curve are the theoretically-derived distributions for the equivalent initial flaw size given by Eq. (12). The solid and dashed curves are for β and β_γ for γ= 95% confidence level, respectively.

Considering the theoretical EIFS distribution represented by the solid curve of Fig. (23) and the crack growth master curve represented by curve 1 of Fig. (13), we can demonstrate the crack damage statistics in service and the economical life of the hypothetical component using the analytical methodology developed in the previous sections.

61

Table 4   TTCI AND EIFS FOR WPF AND XWPF DATA SETS

WPF		XWPF	
TTCI (Flight Hours)	EIFS (Mils)	TTCI (Flight Hours)	EIFS (Mils)
6560	3.281	6111	0.903
9310	1.656	6140	0.893
10631	1.232	7432	0.566
10888	1.165	7535	0.547
11348	1.057	7723	0.514
11630	0.997	8422	0.412
12051	0.914	8528	0.399
13373	0.704	8611	0.389
13540	0.682	8905	0.357
13753	0.655	8967	0.350
13765	0.653	9074	0.339
13783	0.651	9080	0.339
13936	0.633	9312	0.317
14106	0.613	9955	0.265
14124	0.611	10244	0.245
14129	0.610	10452	0.231
14256	0.596	10894	0.206
14347	0.586	11030	0.199
14400	0.580	11035	0.199
14433	0.577	11036	0.199
15492	0.477	11326	0.185
15600	0.468	11418	0.181
15807	0.451	11472	0.178
16079	0.430	11557	0.175
16087	0.429	11695	0.169
16719	0.386	11700	0.169
16741	0.384	11710	0.168
16752	0.384	11907	0.160
16979	0.369	12042	0.155
17043	0.365	12114	0.153
17056	0.365	12303	0.146
17396	0.345	12387	0.143
17556	0.336	12500	0.139
18139	0.305	12623	0.136
18696	0.279	12660	0.134
20140	0.224	13325	0.116
20449	0.214	16004	0.067
23767	0.134		

Without inspection and repair maintenance procedures, the probability $p(i,\tau)$, that a detail will have a crack size larger than x_1 at any service time τ is computed from Eq. (35) . The average percentage of details $\bar{L}(\tau)/N^*$, see Eqs. (26) and (28) , having a crack size larger than x_1 is plotted in Figure (25) as a function of service time τ in flight hours. In Figure (25) , the ordinate $\bar{L}(\tau)/N^*$, denoted by the average percentage of crack exceedance, is the average number of details, while the abscissa is the crack size x_1. For instance, at 8,000 flight hours, the average percentage of fastener holes having a crack size exceeding 0.03" and 0.05" are 2% and 0.8%, respectively. As expected, the average percentage of crack exceedance, $\bar{L}(\tau)/N^*$, increases as the service time τ increases. Curves in Figure (25) are referred to as the average crack exceedance curves.

Table 5 Model Parameters Used for Methodology
Demonstration for EIFS

	WPF *	XWPF **
b	1.2165	1.26
Q	0.9247×10^{-3}	0.2328×10^{-2}
α	4.8634	5.6995
β	14,957 hours	11,372
ϵ	1,312	0
$\beta\gamma$	14,240	10,864

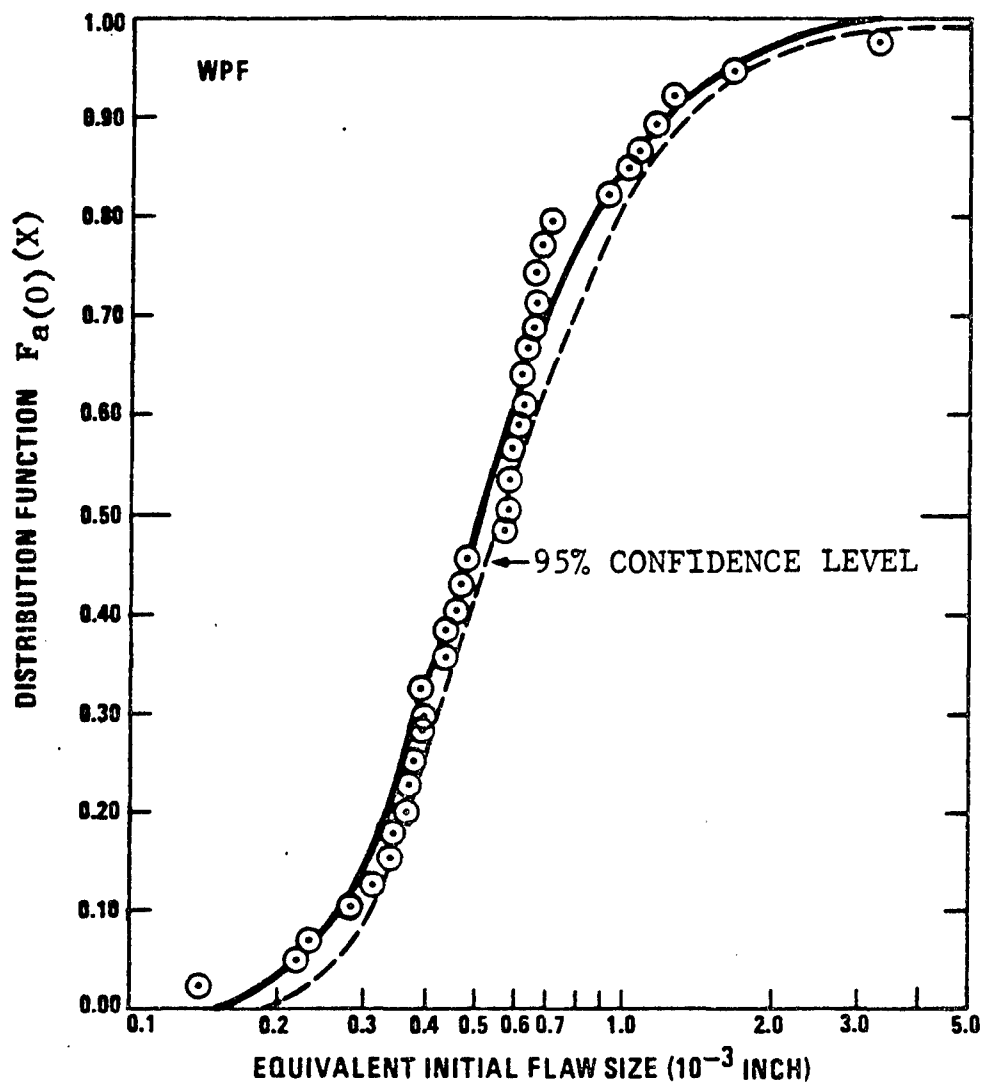*No load transfer

**15% load transfer

Figure 23   Cumulative Probability Versus EIFS for
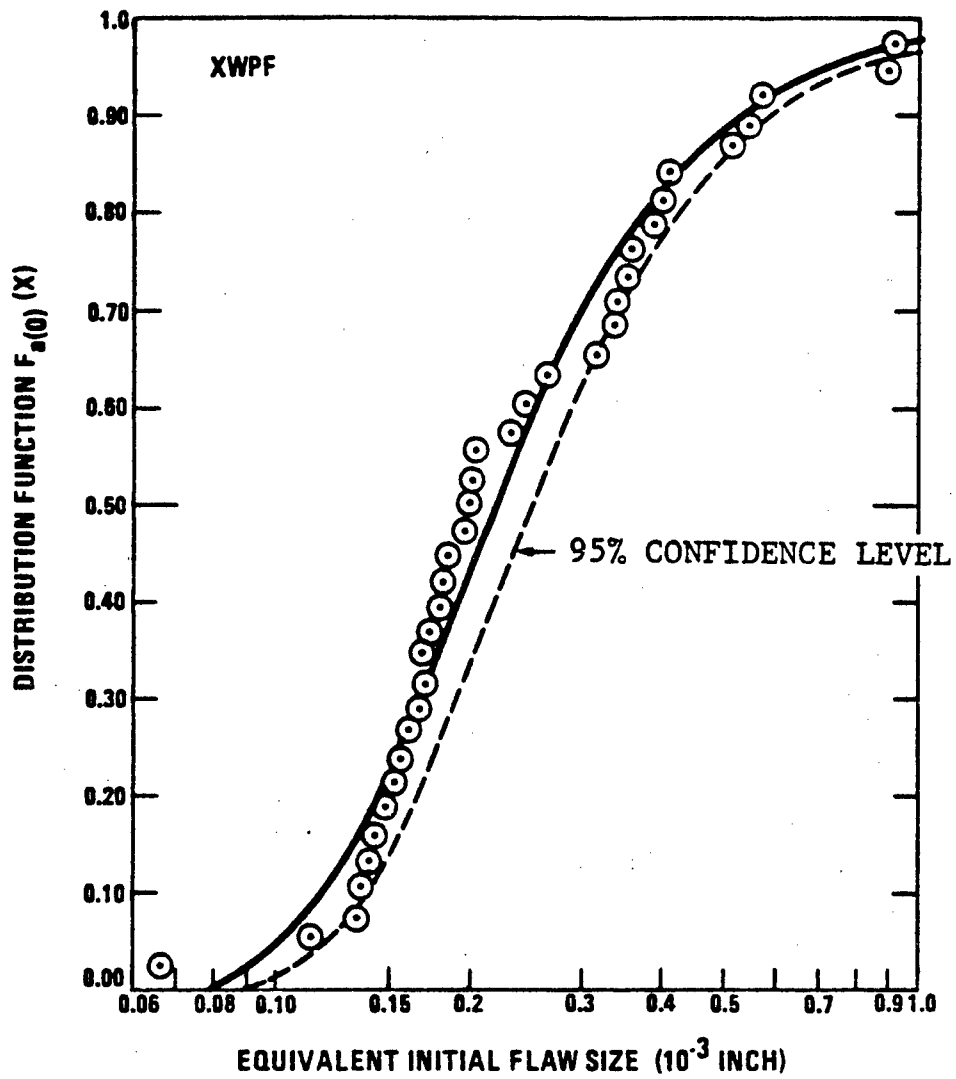Weibull Compatible Distribution (WPF)

64

Figure 24   Cumulative Probability Versus EIFS for
Weibull Compatible Distribution (XWPF)

As discussed in Section V, the number of details having a crack size exceeding x_1 at any service time τ is a statistical variable following the Binomial distribution represented by Eq. (22). When the number of details (fastener holes) is large, the Binomial distribution of Eq. (22) can be approximated by the normal distribution with the mean and the variance given by Eqs. (24) and (25), respectively. As a result, Figure (25), represents an approximation of the percentage of fastener holes having a crack size exceeding x_1 (an abscissa) with 50% probability.

From the standpoint of the durability requirement and the economic life criteria, the central measure of the crack exceedance, i.e., exceedance with 50% probability, as shown in Figure (25), may not be conservative enough, although it provides information for estimating the average repair cost as will be described later. Hence, it is desirable to compute the number (or percentage) of details having a crack size exceeding x_1 at any service time τ with certain probability, in particular the low probability of exceedance. This objective can be accomplished using either the Binomial distribution given by Eq. (22) or the normal distribution for approximation.

For simplicity in presenting the numerical results, the service time of $\tau=8000$ flight hours is considered. The percentage of cracks, $L(\tau)/N^*$, Eqs. (26) and (28), exceeding the crack size x_1 (abscissa) is plotted in Figure (26) for various exceedance probabilities. For instance, at $\tau=8,000$ flight hours, the probability is 0.1 that 3.764% of the total cracks will exceed the crack size 0.03" as indicated by an open circle. Similarly, the probabilities are 0.5 and 0.8, respectively, that 2% and 0.803% of the total cracks will exceed 0.03" (see open circles). It is further observed from Figure (26) that the probabilities are 0.1, 0.5 and 0.8, respectively, that 1.97%, 0.8% and 0.06% of the total cracks will exceed 0.05" as shown by the solid circles. Figure (26) indicates that as the probability of exceedance decreases, the percentage of cracks exceeding a certain crack size x_1 increases as expected.

The numerical results presented in Figures (25) and (26) for the crack exceedance with certain probability does not account for the confidence level, since the value of β representing the characteristic time to crack initiation is used in the analyses (see Table 5). When a confidence level γ is further considered, then β_γ should be used. The value of β_γ for $\gamma=95\%$ confidence level given in Table 5 is now used in the analyses. Numerical results are presented in Figures (27) and (28) which correspond to a 95% confidence level. For instance, it is observed from Figure (28) that at 8,000 flight hours, the probability is 0.1 that 4.51% of the total cracks will exceed 0.03" with 95% confidence level as indicated by a circle. In other words, at 8,000 flight hours, 4.51% of the total cracks will exceed 0.03" with 0.1 probability and 95% confidence.
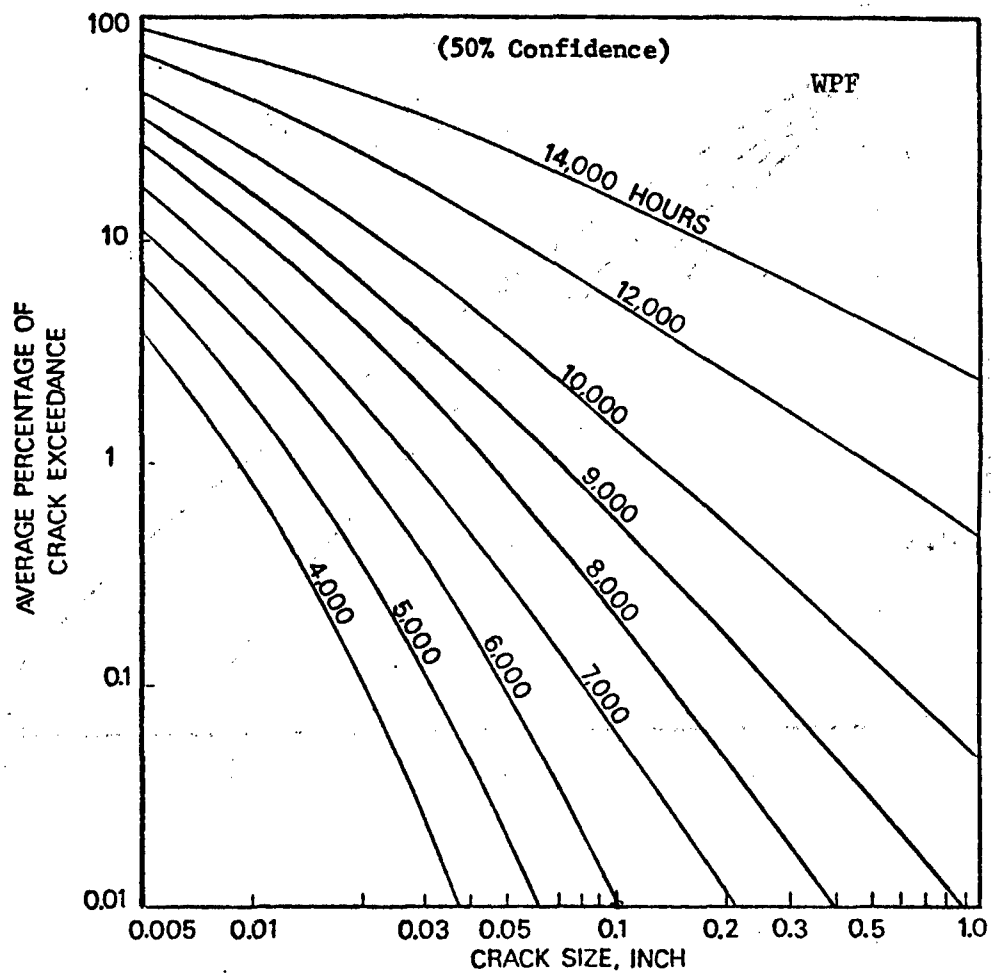
66

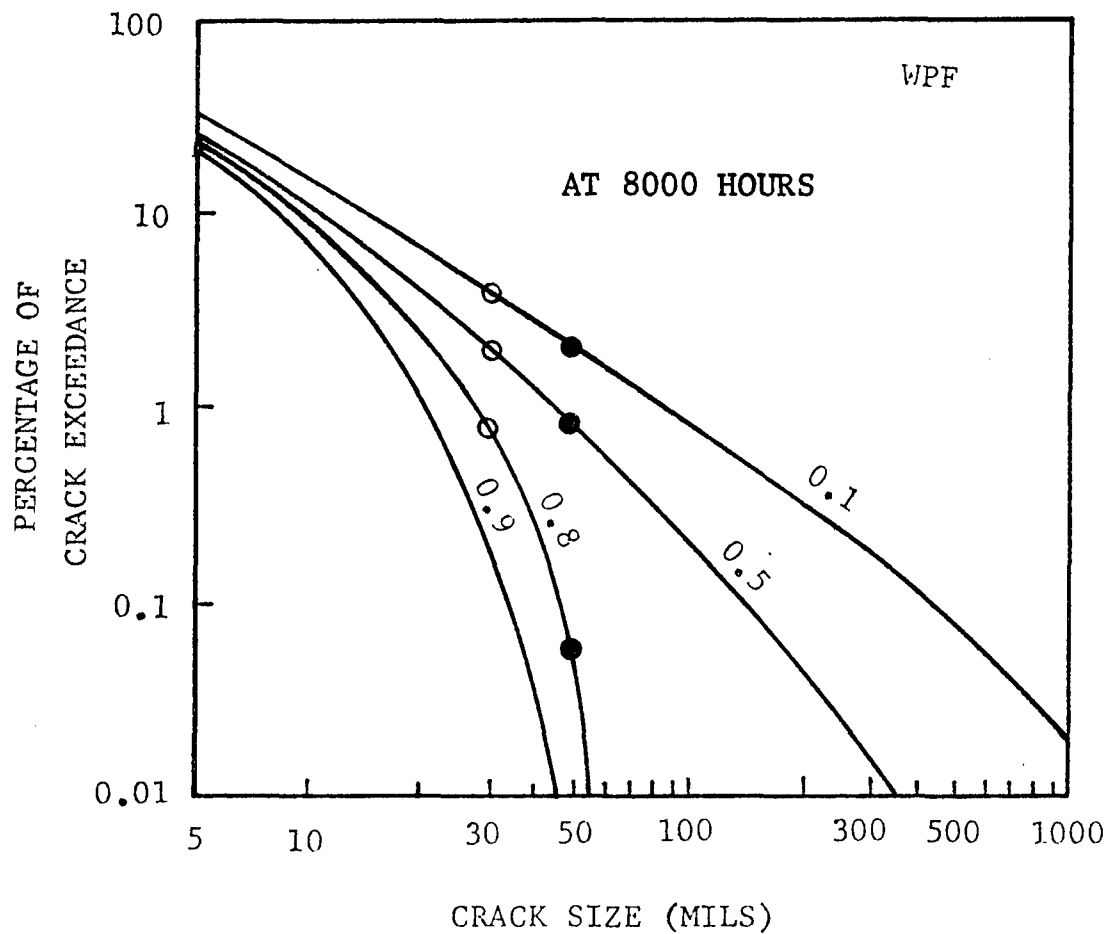Figure 25 Crack Exceedance Curves for 50% Confidence Level

Figure 26 Crack Exceedance and Reliability
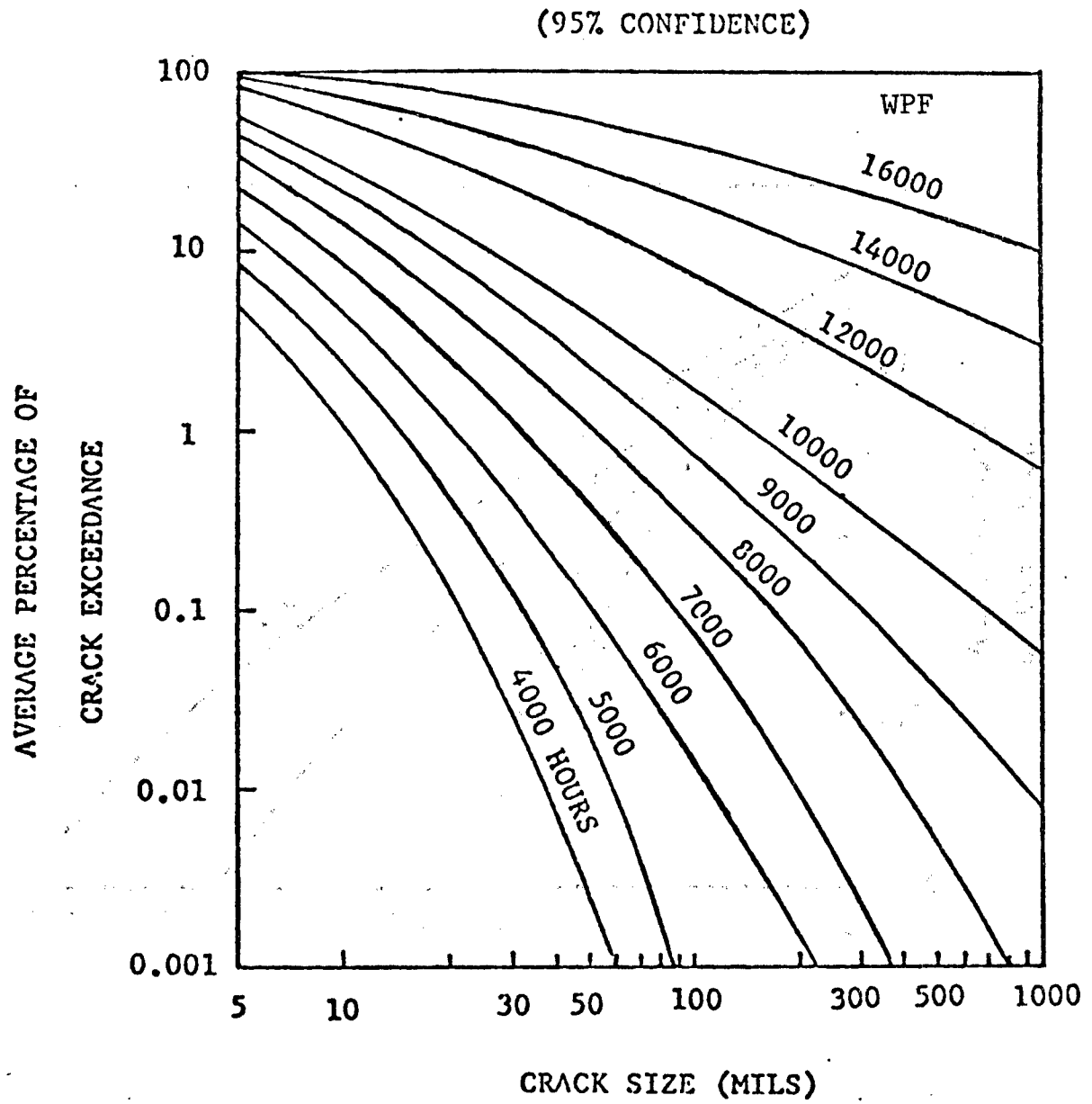for 50% Confidence Level

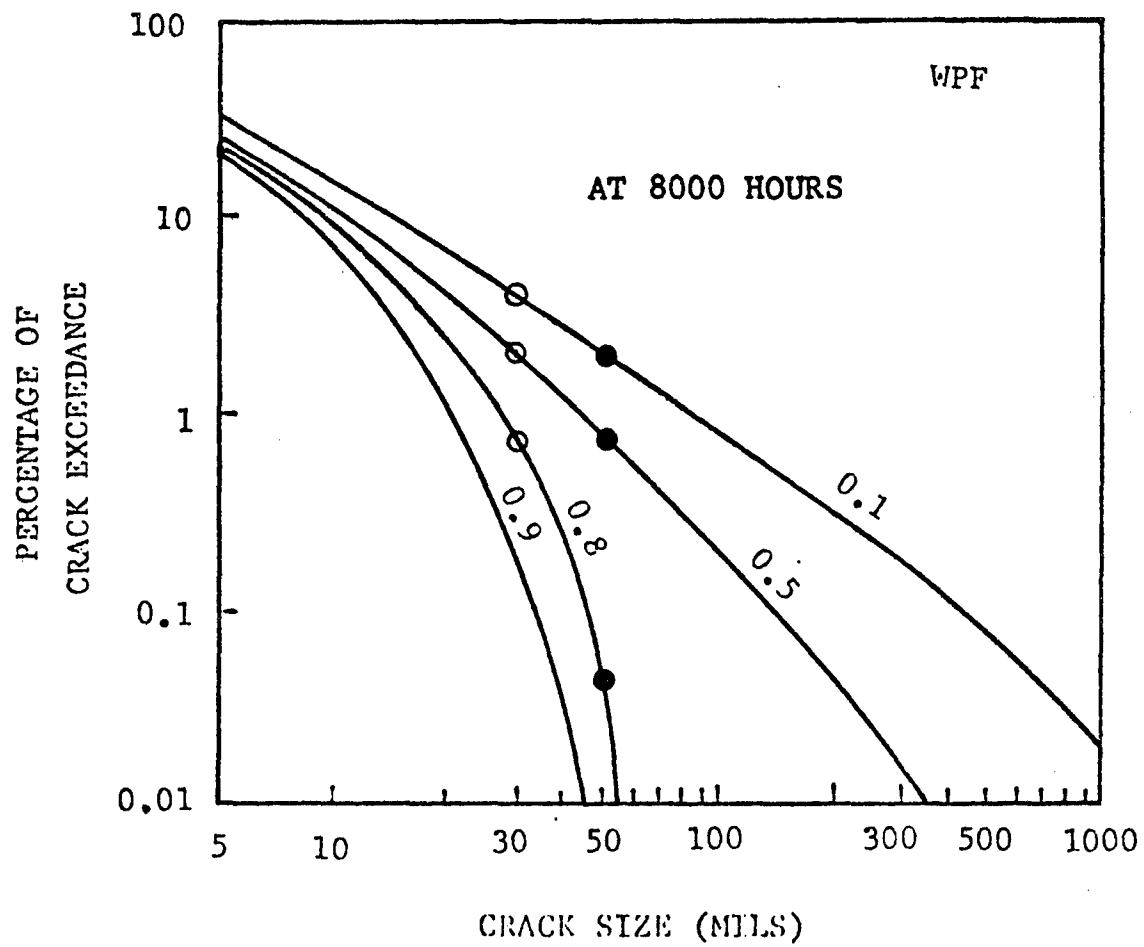Figure 27 Crack Exceedance Curves (95% Confidence)

Figure 28 Crack Exceedance and Reliability (95% Confidence)

A comparison between Figs. (25) and (27) indicates that the average percentage of cracks exceeding any given crack size x_1 increases when a higher confidence level is considered. Further comparison between Figs. (26) and (28) shows that the percentage of cracks exceeding any given crack size x_1 with any given probability increases as a higher level of confidence is taken into account

The economic repair crack size, a_e, is arbitrarily defined as the maximum crack size below which the least expensive repair procedure can be used i.e., reaming the fastener hole to the next hole size. For the above definition, a_e is usually between 0.03" and 0.05" depending on the location and the fastener hole size. If 0.03" is used as the economic repair crack size, then the average percentage of details having a crack exceeding a_e=0.03" is plotted as a function of the service time as shown by Curve 1 of Figure (29). This curve, in fact, can be constructed from Figure (25) by drawing a vertical line passing through 0.03". The average percentage of cracks exceeding a_e is also the percentage of cracks exceeding a_e with 0.5 probability. Hence, Curve 1 is referred to as the exceedance curve for the economic repair crack size a_e with probability 0.5. The exceedance curves for the economic repair crack size with probability 5% and 10%, respectively, are also computed and plotted in Figure (29) as dashed curves. It is observed from the figure that Curve 1 increases rapidly after 8,000 flight hours. If 5% of the cracked details exceeding 0.03" with probability 50% is considered as an economical limit, then the possible economic life is observed to be 9,400 flight hours.

When the economic repair crack size a_e is 0.05", one can construct in a similar fashion the exceedance curve with probabilities 50% (Curve 1), 10% (dashed curve) and 5% (dashed curve) as shown in Figure (30). The possible economic life in this case is found to be 10,500 flight hours if 5% of the cracks exceeding 0.05" with probability 50% is considered as an economic limit.

In the same manner, one can also construct the exceedance curves for the economic repair crack size a_e=0.03" with probabilities 0.5 (solid curve), 0.1 (dashed curve) and 0.05 (dashed curve) as a function of service life when a confidence level of γ=95% is taken into account. The results are displayed in Figure (31).

In order to demonstrate the effect of inspection and repair maintenance procedures on the economic life, an NDI procedure having a high resolution capability is assumed. The functional form for the detection probability $F_D(x)$ given by Eq. (51) is used with a_1=0.01", a_2=0.1" and m=0.5. Such a detection probability $F_D(x)$ is plotted in Figure (32) as a function of the crack size x.

The cost formulation for the economic life criteria as represented by Eq. (30) is based on the average (expected) maintenance cost, including the average costs of inspection and repair. Since the average cost of repair is related to the average
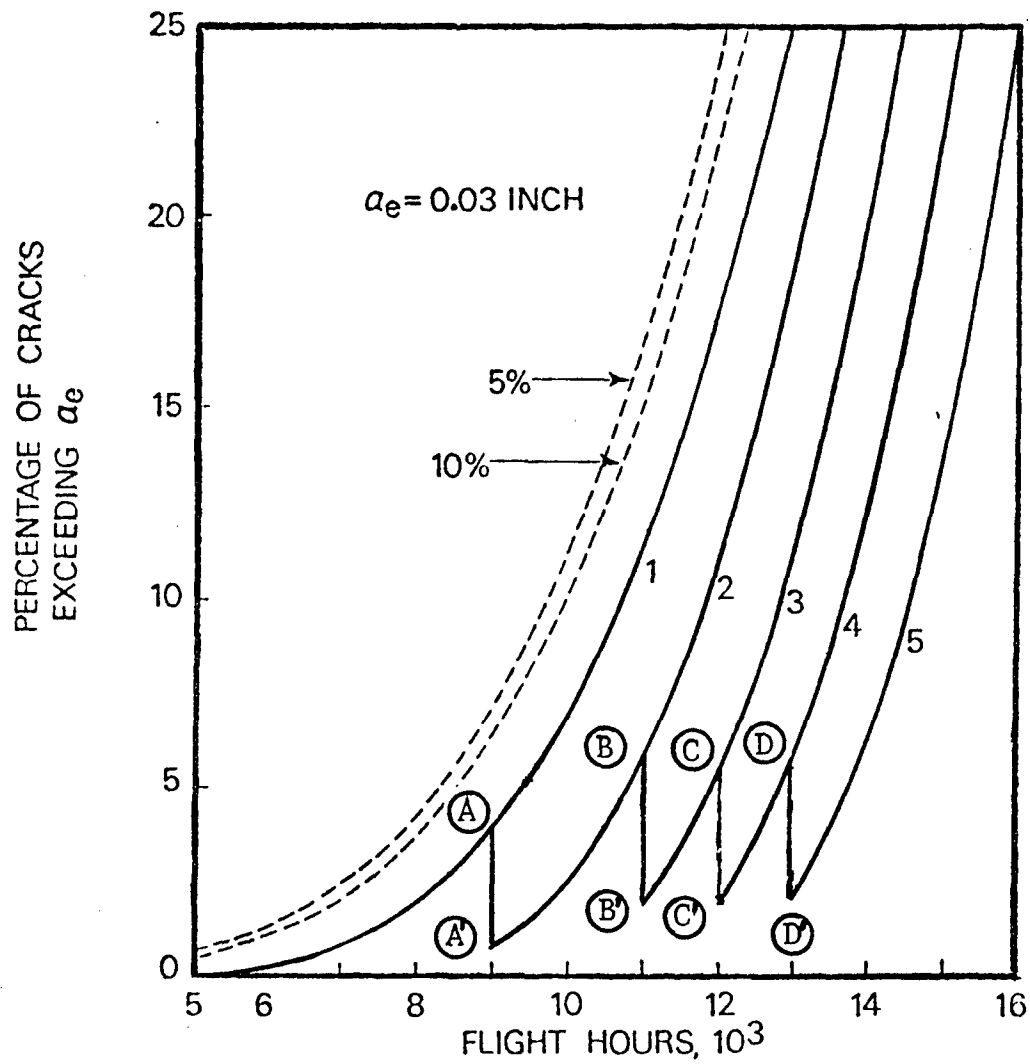
71

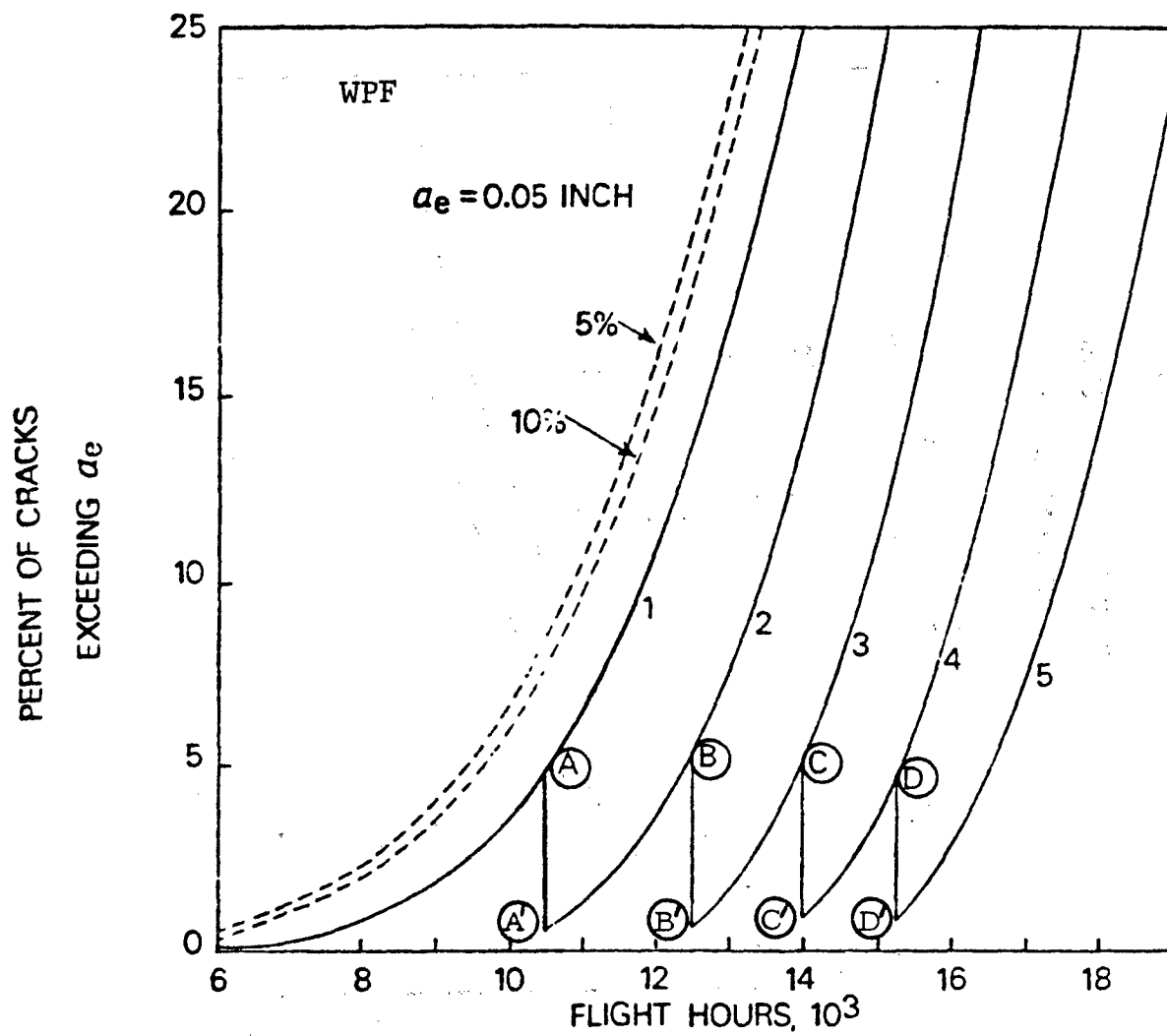Figure 29 Crack Exceedance Versus Service Time Showing the Effects of Repairs (a_e = 0.03")

Figure 30 Crack Exceedance Versus Service Time Showing the Effects of Repairs ($a_e = 0.05"$)
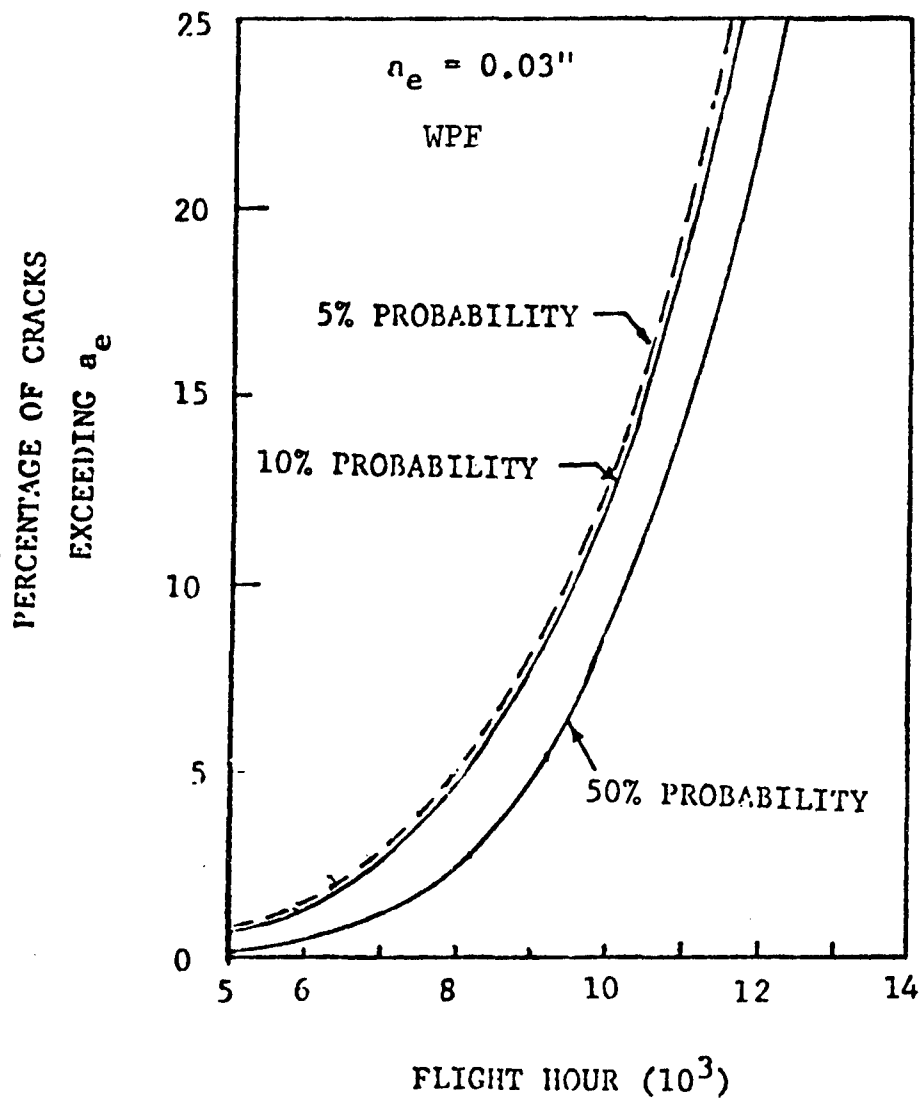
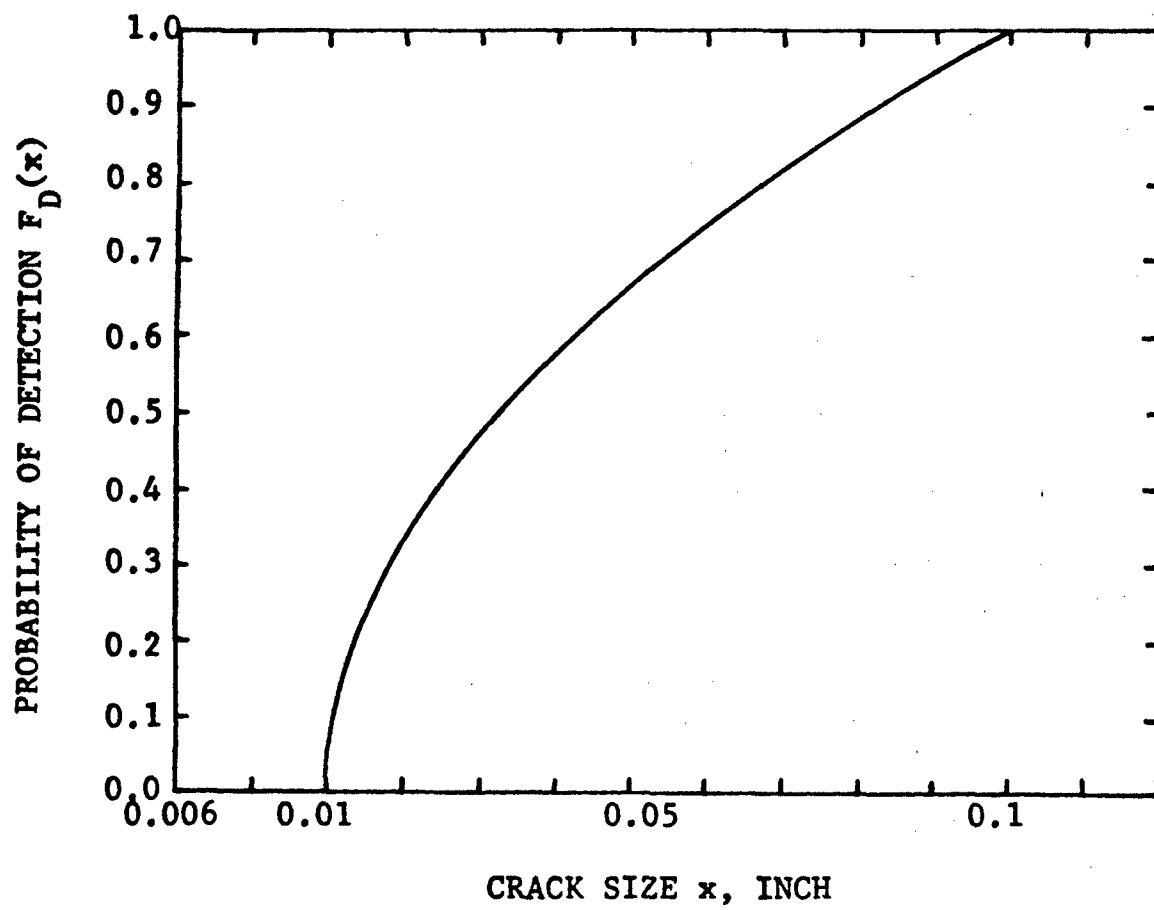Figure 31 Crack Exceedance Versus Service Time (95% Confidence)

Figure 32   Crack Detection Probability $F_D(x)$

75

number of detected cracks in various size ranges, $g_j(x)dx$, the effect of the maintenance procedures on the economic life will be investigated on the basis of the average crack exceedance curve (i.e., with 50% exceedance probability) as shown by Curve 1 of Figures (29) and (30) with 50% confidence level.

For the case where $a_e=0.03"$, Figure (29) , the first inspection and repair is performed at $T_1=9500$ flight hours. The resulting average exceedance curve (with 50% probability) is obtained by use of Eqs. (43) to (45) and plotted in Figure (29) as Curve 2. Owing to the repair procedures, there is a sudden drop from Ⓐ to Ⓐ' (i.e., Curve 1 to Curve 2) at T_1 as indicated in Figure (29) . According to Curve 2 of Figure (29) , the possible economic life (5% allowable exceedance over a_e with 50% probability as defined previously) is now extended to 11,000 flight hours.

If the second inspection and repair is further performed at $T_2=11,000$ flight hours, then the resulting average exceedance curve is computed using Eqs. (43) to (45) and plotted in Figure (29) as Curve 3. The economic life is thus extended to 12,000 flight hours. In a similar manner, additional inspections and repair maintenance procedures are performed, respectively, at $T_3=12,000$ and $T_4=13,000$ flight hours. The resulting average crack exceedance curves are plotted in Figure (29) as Curves 4 and 5, respectively. It is observed from Figure (29) that there is a sudden drop of the average percentage of crack exceedance at each inspection and repair maintenance, which is exactly the average percentage of cracks repaired (or detected) during each maintenance.

Thus, if the durability critical component is subjected to a scheduled inspection and repair maintenance procedure at T_1, T_2, T_3, and T_4, then the average crack exceedance curve will follow Curve 1 up to Ⓐ and then follows the path Ⓐ→Ⓐ'→ Ⓑ→Ⓑ'→Ⓒ→Ⓒ'→Ⓓ→Ⓓ'→ Curve 5.

For the case where the economic repair crack size is 0.05", the inspection and repair maintenance procedures are performed at $T_1=10,500$, $T_2=12,500$, $T_3=14,000$ and $T_4=15,250$ flight hours, respectively. The resulting average crack exceedance curves are presented in Figure (30). If, for instance, the inspection and repair maintenance are performed at T_1 and T_2 only, then the resulting average crack exceedance curve will follow Curve 1 up to Ⓐ and then follows the path Ⓐ→Ⓐ'→Ⓑ→Ⓑ'→Curve 3.

The average percentage of details having a crack size larger than z_1 which are repaired at T_j, denoted by $q_j(z_1,\infty)$ (see Eq. (49)), is plotted in Figure (33) for j=1,2,3, and 4 for the case where $a_e=0.03"$. Curves in Figure (33) are referred to as the average exceedance curves for the repaired cracks. For instance, one percent of the details having a crack size larger than 0.1" is expected to be repaired at $T_1=9,500$ flight hours as indicated by a circle on Curve 1 of Figure
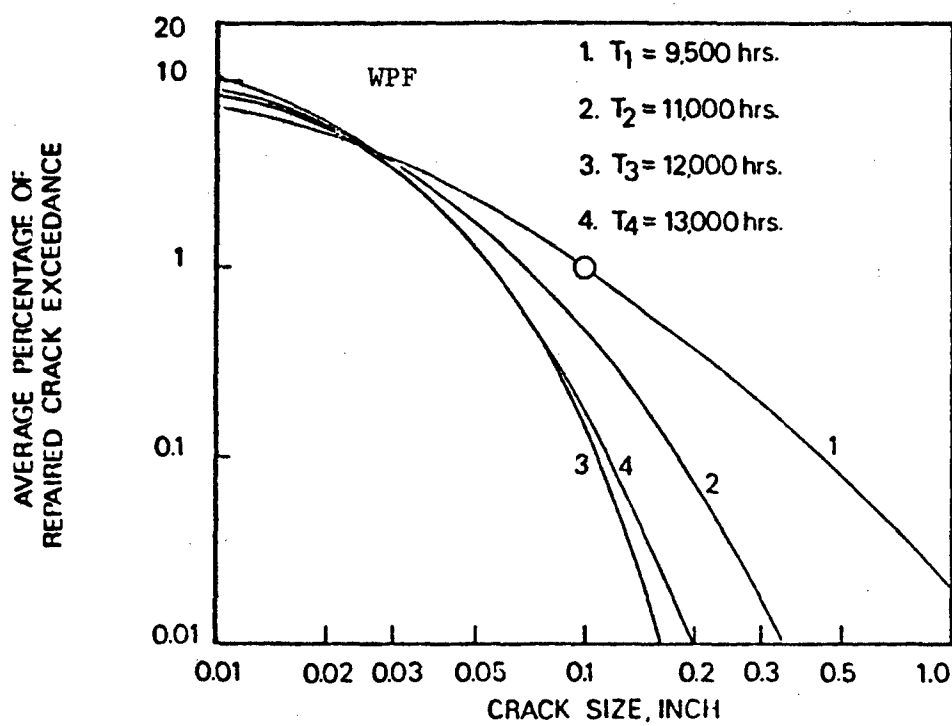
Figure   33   Average Exceedance Curves for Repaired Details

(33) . The exceedance curves depicted in Figure (33) indicate, on the average, the number of details as well as their corresponding crack size to be repaired during each inspection and repair maintenance. These curves contain the information needed for the computation of the average repair cost represented by Eq. (30) , since $g_j(x)=-dq_j(x,\infty)/dx$, see Eq. (50).

It is observed from Figures (29) and (30) that the effect of inspection and repair maintenance procedures on the economic life is small even using a high resolution NDI procedure. It is found that for other NDI techniques with poor detection capability, the effect is practically minimal. The reasons are given as follows: (i) To extend the economic life, the NDI procedure should be able to detect with high probability the crack size below the economic repair crack size 0.03" in order to prevent cracks from propagating beyond a_e. Current NDI techniques may not achieve such a capability with a comparable cost, and (ii) As soon as the crack exceedance curve over a_e starts to increase rapidly [see Figures (29) and (30) the majority of the crack population is already in the vicinity of a_e, and hence a significant extension of the economic life implies an extensive repair such that the cost of repair may be uneconomical.

SECTION IX

CONCLUSIONS AND DISCUSSIONS

A durability analysis methodology for aircraft structures has been developed. The durability analysis is based on the statistical approach which allows for the determination of a possible economic life using one of the following criteria: (i) the percentage of crack population exceeding the economic repair crack size with certain probability and confident levels, (ii) the ratio of the cost of maintenance, including the costs of inspection and repair, to the initial cost. Although the inspection and repair maintenance procedures have a significant impact on the reliability and safety of aircraft structures, its effect in extending the economic life is shown to be limited.

In the present durability analysis, the input initial fatigue quality can either be the time to crack initiation (TTCI) or the equivalent initial flaw size (EIFS). The three-parameter Weibull distribution is used for the statistical distribution of the TTCI. The statistical distribution, which is derived from that of the TTCI using a particular crack propagation law, is suggested for the distribution of the EIFS. Such a derived distribution possesses the physical meaning for the fatigue wear-out process and it is shown to fit the EIFS better than other distributions.

The CGR computer code is used to calculate the master curve (crack growth damage) in each stress region of the entire durability critical component in service. The binomial distribution is used to compute the statistical distribution of the number of fastener holes having a crack in any size range. The crack exceedance with certain probability and confidence is then estimated as a function of the service time. The relaibility of the durability critical component in service is also obtained. A scheduled (nonperiodic) inspection and repair maintenance procedure is considered, in which the statistical uncertainty of the NDI procedure and crack growth damage of the repaired details are taken into account. The average cost of maintenance, including the costs of inspection and repair, is then formulated as a function of service time.

While the second criterion for determining the possible economic life has been formulated and relevant statistics, such as the crack exceedance for the repaired cracks, have been obtained, it is not demonstrated numerically. This is due to a lack of realistic information of the field inspection cost as well as the cost of repairing one crack in a certain size range. It appears that the first criterion of the economic life may be more appropriate at the preliminary design stage for the durability requirements.

Although the present approach and formulation are demonstrated using the example of fastener holes, they are applicable to cracks which occur at radii or shear webs with holes, as long as the crack growth master curve is given or computed.

There are circumstances where the usage or missions of aircraft may deviate significantly at certain service time of its design life. Such a usage change can easily be taken care of in the present durability analysis. For instance, when the usage change occurs at t_c flight hours, then the new crack growth master curve, which corresponds to the new usage, should be used in the analysis after t_c and all the analysis procedures remain the same.

Although the reliability of the durability critical component has been formulated within the framework of the durability analysis, it is not demonstrated numerically. It will be investigated in the Phase II effort whether the reliability problem is important in the durability requirement.

In the present approach, the crack growth master curve under spectrum loading is considered deterministic, i.e., the statistical variability of crack growth parameters is neglected. This may be a reasonable approximation, since the statistical variability of the crack growth parameters is smaller than that of the equivalent initial flaw size, and hence the statistical variability of the crack exceedance curves (or economic life) is essentially due to that of the EIFS. Such an approximation simplifies the entire analysis tremendously. The effect of the statistical variability of the crack growth damage on the economic life of aircraft structures will be investigated in the Phase II effort.

It should be emphasized that the statistical variability of the crack growth damage is important and should be accounted for in the damage tolerant analysis. In the damage tolerant analysis, the initial flaw size is specified and, hence, the statistical uncertainty in structural life prediction is essentially contributed to by the uncertainty of the crack growth damage when the service loads are idealized as spectrum loadings.

APPENDIX

DERIVATION OF PROBABILITY DENSITY FUNCTION $f_{a(T_j+t)}(x)$

(A) SPECIAL CASE

In order to facilitate the derivation, the crack growth rate equation represented by Eq. (8) will first be used for simplicity. Eq. (8) will only serve as a convenient vehicle to explain the mathematical derivation. The general results without the limitation of Eq. (8) will be described later.

Integration of Eq. (8) from t_1 to t_2 yields

$$a(t_1) = \frac{a(t_2)}{[1 + a^c(t_2)\,cQ(t_2-t_1)]^{1/c}} = W[a(t_2), t_2-t_1] \qquad \text{(A-1)}$$

or

$$a(t_2) = \frac{a(t_1)}{[1 - a^c(t_1)\,cQ(t_2-t_1)]^{1/c}} = W^{-1}[a(t_1), t_2-t_1] \qquad \text{(A-2)}$$

in which $t_1 \leq t_2$, $W^{-1}[.]$ is the inversed function of $W[.]$ and

$$c = b - 1 \qquad \text{(A-3)}$$

For $t_1 = 0$ and $t_2 = t$, Eqs. (A-1) and (A-2) reduce to

$$a(0) = \frac{a(t)}{[1 + a^c(t)\,cQt]^{1/c}} = W[a(t), t]$$

$$\text{(A-4)}$$

$$a(t) = \frac{a(0)}{[1 - a^c(0)\,cQt]^{1/c}} = W^{-1}[a(0), t]$$

where a(0) is the equivalent initial flaw size.

Eqs. (A-1) to (A-4) will serve as the transfer functions to transform the distribution function of the EIFS into that of the flaw size at any service time.

It should be mentioned that the values of Q and b (or c=b-1) vary from one stress region to another. However, the values of Q and b (or c=b-1) appearing in the distribution of EIFS are obtained from one data set of coupon specimens under the same loading spectra but at a particular maximum stress level. As a result, Q and c appearing in this appendix should be written as Q_i and c_i except Eqs. (A-7) and (A-8). Because of simplicity in notation, the subscript i has been dropped and they should not be confused with Q and c for the EIFS.

(1) First Service Interval $(0, T_1)$

In the first service interval, the entire initial crack population is subjected to propagation following Eq. (A-4).

Let $F_{a(t)}(x)$ and $f_{a(t)}(x)$ be the cumulative distribution function and the probability density function, respectively, of the crack size a(t) at t flight hours in the first service interval. Then both $F_{a(t)}(x)$ and $f_{a(t)}(x)$ can be derived from those of the initial flaw size $F_{a(0)}(x)$ and $f_{a(0)}(x)$, respectively, through the transformation of Eq. (A-4) as follows:

$$F_{a(t)}(x) = P[a(t) \leq x] = P[a(0) \leq y_1(t)] = F_{a(0)}[y_1(t)]$$

or

$$F_{a(t)}(x) = F_{a(0)}[y_1(t)] \qquad (A-5)$$

in which $y_1(t)$ is a value of a(0) corresponding to a value x of a(t). They are related through Eq. (A-4) by replacing a(0) and a(t), respectively, by $y_1(t)$ and x.

$$y_1(t) = \frac{x}{[1 + x^c cQt]^{1/c}} = W(x,t) \qquad (A-6)$$

Substituting Eq. (12) into Eq. (A-5), one obtains

$$F_{a(t)}(x) = \exp\left\{-\left[\frac{y_1^{-c}(t)-a_0^{-c}-cQ\epsilon}{cQ\beta}\right]^{\alpha}\right\} \; ; \qquad y_1(t) \leq x_u \qquad \text{(A-7)}$$

$$= 1 \qquad \qquad \qquad ; \qquad y_1(t) > x_u$$

in which $y_1(t)$ is a function of x given by Eq. (A-6) and x_u is given by Eq. (13).

The probability that the crack size, in the ith stress region at $\tau = t$ service hours, will exceed x_1 is given by

$$p(i,\tau) = 1 - F_{a(\tau)}(x_1)$$

$$= 1 - \exp\left\{-\left[\frac{y_1^{-c}(\tau)-a_0^{-c}-cQ\epsilon}{cQ\beta}\right]^{\alpha}\right\} \; ; \qquad y_1(\tau) \leq x_u \qquad \text{(A-8)}$$

$$= 0 \qquad \qquad \qquad ; \qquad y_1(\tau) > x_u$$

in which Eq. (A-7) has been used and $y_1(\tau)$ is related to x_1 through Eq. (A-4) as

$$y_1(\tau) = \frac{x_1}{[1 + x_1^c cQ\tau]^{1/c}} = W[x_1,\tau] \qquad \text{(A-9)}$$

As expected, $p(i,\tau)$ obtained in Eq. (A-8) is identical to that derived in the text in which no inspection and repair maintenance is scheduled.

The transformation of the probability density function from $f_{a(0)}(x)$ to $f_{a(t)}(x)$ can be found in any text book on probability and statistics,

$$f_{a(t)}(x) = f_{a(0)}[y_1(t)]J_1(t) \qquad \text{(A-10)}$$

in which $y_1(t) = W(x,t)$ is related to x through Eq. (A-6) and

$$J_1(t) = \frac{dy_1(t)}{dx} = \frac{dW(x,t)}{dx} = \frac{1}{\left[1+x^c cQt\right]^{\frac{1}{c}+1}} \qquad \text{(A-11)}$$

where Eq. (A-6) has been used.

83

At the end of the first service interval, i.e., $t = \tau_1 = T_1$, the probability density function of the crack size $a(T_1)$ given by Eq. (A-10) becomes

$$f_{a(T_1)}(x) = f_{a(0)}[y_1(\tau_1)]J_1(\tau_1) \qquad \text{(A-12)}$$

in which it follows from Eqs. (A-6) and (A-11) that

$$y_1(\tau_1) = \frac{x}{\left[1+x^c cQ\tau_1\right]^{1/c}} = W(x,\tau_1)$$

$$J_1(\tau_1) = \frac{1}{\left[1 + x^c cQ\tau_1\right]^{\frac{1}{c}+1}} = \frac{dW(x,\tau_1)}{dx} \qquad \text{(A-13)}$$

The component is then subjected to inspections. The probability, $g_1(x)dx$, of detecting a crack in the size range $(x, x+dx)$ at $t = \tau_1 = T_1$ is therefore

$$g_1(x)dx = F_D(x)f_{a(T_1)}(x)dx \qquad \text{(A-14)}$$

in which $f_{a(T_1)}(x)$ is given by Eq. (A-12) and $F_D(x)$ is the probability of detecting a crack size x.

The probability of detecting a crack in the size range z_1 and z_2 during the first inspection, denoted by $q_1(z_1, z_2)$ is

$$q_1(z_1, z_2) = \int_{z_1}^{z_2} g_1(x)dx = \int_{z_1}^{z_2} F_D(x)f_{a(T_1)}(x)dx \qquad \text{(A-15)}$$

and the probability of detecting a crack of any size at T_1, indicated by $G(1)$, is

$$G(1) = q_1(0, \infty) = \int_0^\infty g_1(x)dx = \int_0^\infty F_D(x)f_{a(T_1)}(x)dx \qquad \text{(A-16)}$$

84

(2) Second Service Interval (T_1, T_2)

It is assumed that when a crack is detected, it is repaired. Thus, after the first inspection and repair performed at T_1, the details (fastener holes) consist of two different populations; (1) the population where cracks have been detected and repaired, referred to as the renewal details, and (ii) the population where the cracks have not been detected and hence details have not been repaired. It is further assumed that the crack size distribution of the renewal details is identical to that of the equivalent initial flaw size a(0). Consequently, the probability density function of the crack size $a(T_1^+)$ right after the first inspection and repair, denoted by $f_{a(T_1^+)}(x)$, consists of two parts

$$f_{a(T_1^+)}(x) = G(1) f_{a(0)}(x) + F_D^*(x) f_{a(T_1)}(x) \qquad (A-17)$$

in which

$$F_D^*(x) = 1 - F_D(x) \qquad (A-18)$$

is the probability of missing a crack size x during inspection, and G(1) and $f_{a(T_1)}(x)$ are given by Eqs. (A-16) and (A-12), respectively.

It is obvious that the first term in Eq. (A-17) represents the contribution from the renewal (repaired) population with probability G(1), while the second term is contributed by the old population (which survives the inspection with probability $F_D^*(x)$).

Substitution of Eq. (A-13) into Eq. (A-12) and then into Eq. (A-17) yields the following expression for $f_{a(T_1^+)}(x)$.

$$f_{a(T_1^+)}(x) = G(1) f_{a(0)}(x) + F_D^*(x) f_{a(0)} \left[\frac{x}{\left(1+x^c cQ\tau_1\right)^{1/c}} \right] \frac{1}{\left[1+x^c cQ\tau_1\right]^{\frac{1}{c}+1}} \qquad (A-19)$$

In the second service interval (T_1, T_2), the two different crack populations given by Eq. (A-19) are subjected to propagation in service. Let the probability density function of the crack size $a(\tau)$ at any service time $\tau = T_1 + t$ in (T_1, T_2) be denoted by $f_{a(T_1+t)}(x)$. Then, $f_{a(T_1+t)}(x)$ can be obtained from $f_{a(T_1^+)}(x)$

given by Eq. (A-19) through the transformation of Eq. (A-1) with $t_2 = \tau_1 + t$, $t_1 = \tau_1$ $\left[T_1 = \tau_1, \text{ see Fig. (1)}\right]$, i.e.,

$$a(\tau_1) = \frac{a(\tau_1 + t)}{\left[1 + a^c(\tau_1 + t) cQt\right]^{1/c}} = W[a(\tau_1 + t), t] \qquad (A-20)$$

Following the same procedures described above by: (i) replacing the argument of x in Eq. (A-19) by $y_1(t) = W(x,t)$ [Eq. (A-6)] and (ii) multiplying Eq. (A-19) by $J_1(t)$, one obtains the following expression for $f_{a(T_1+t)}(x)$,

$$f_{a(T_1+t)}(x) = G_1(1) f_{a(0)} [y_1(t)] J_1(t)$$

$$+ F_D^*[y_1(t)] f_{a(0)} [y_1(\tau_1+t)] J_2(\tau_1+t) J_1(t) \qquad (A-21)$$

in which it can be shown that

$$y_1(\tau_1+t) = W(x, \tau_1+t) = \frac{x}{\left[1 + x^c cQ(\tau_1+t)\right]^{1/c}}$$

$$\qquad (A-22)$$

$$J_2(\tau_1+t) = \frac{dy_1(\tau_1+t)}{dx} = \left[\frac{1+x^c cQt}{1+x^c cQ(t+\tau_1)}\right]^{\frac{1}{c}+1}$$

At the end of the second service interval, i.e., $T_2 = T_1 + \tau_2$ (or $t = \tau_2$), the probability density function of the crack size $a(T_2)$ follows from Eq. (A-21) as

$$f_{a(T_2)}(x) = G(1) f_{a(0)} [y_1(\tau_2)] J_1(\tau_2)$$

$$+ F_D^*[y_1(\tau_2)] f_{a(0)} [y_1(\tau_1+\tau_2)] J_2(\tau_1+\tau_2) J_1(\tau_2) \qquad (A-23)$$

The probability of detecting a crack in the size range (x,x+dx) at T_2, denoted by $g_2(x)dx$, is

$$g_2(x)\,dx = F_D(x)\,f_{a(T_2)}(x) \qquad \text{(A-24)}$$

in which $f_{a(T_2)}(x)$ is given by Eq. (A-23). The probability of detecting a crack size between z_1 and z_2 is

$$q_2(z_1,z_2) = \int_{z_1}^{z_2} g_2(x)\,dx = \int_{z_1}^{z_2} F_D(x)\,f_{a(T_2)}(x)\,dx \qquad \text{(A-25)}$$

and the probability of detecting a crack of any size during the second inspection performed at T_2 is

$$G(2) = q_2(0,\infty) = \int_0^{\infty} F_D(x)\,f_{a(T_2)}(x)\,dx \qquad \text{(A-26)}$$

(3) Third Service Interval (T_2, T_3)

The probability density function of the crack size $a(T_2^+)$ right after the second inspection and repair consists of an additional contribution from the renewal (repaired) population with probability $G(2)$ (i.e., produced by the second inspection and repair),

$$f_{a(T_2^+)}(x) = G(2)\,f_{a(0)}(x) + F_D^*(x)\,f_{a(T_2)}(x) \qquad \text{(A-27)}$$

in which $f_{a(T_2)}(x)$ is given by Eq. (A-23).

Substitution of Eq. (A-23) into Eq. (A-27) yields

$$f_{a(T_2^+)}(x) = G(2)\,f_{a(0)}(x) + G(1)\,F_D^*(x)\,f_{a(0)}\,[y_1(\tau_2)]\,J_1(\tau_2)$$

$$\qquad \text{(A-28)}$$

$$+\ F_D^*(x)\,F_D^*[y_1(\tau_2)]\,f_{a(0)}\,[y_1(\tau_1+\tau_2)]\,J_2(\tau_1+\tau_2)\,J_1(\tau_2)$$

The crack size $a(T_2+t)$ at any service time $T_2+t\ (t \le \tau_3)$ in the third service interval is related to the crack size $a(T_2^+)$ through Eq. (A-1) in which $t_1=T_2$, $t_2=T_2+t$, i.e.,

$$a(T_2) = \frac{a(T_2+t)}{\left[1+a^c(T_2+t)\,cQt\right]^{1/c}} = W\left[a(T_2+t),t\right] \qquad \text{(A-29)}$$

87

The probability density function of $a(T_2+t)$, denoted by $f_{a(T_2+t)}(x)$ is obtained from $f_{a(T_2^+)}(x)$ through the transformation of Eq. (A-29) employing the same procedures described above by: (i) substituting Eqs. (A-22) with $t=\tau_2$ into Eq. (A-28), (ii) replacing x in Eq. (A-28) by $y_1(t)=W(x,t)$ given by Eq. (A-6) and (iii) multiplying Eq. (A-28) by $J_1(t)$; with the results

$$f_{a(T_2+t)}(x) = G(2)f_{a(0)}[y_1(t)]J_1(t) + G(1)F_D^*[y_1(t)]f_{a(0)}$$

$$[y_1(\tau_2+t)]J_2(\tau_2+t)J_1(t) + F_D^*[y_1(t)]F_D^*[y_1(\tau_2+t)]$$

$$f_{a(0)}[y_1(\tau_1+\tau_2+t)]J_2(\tau_1+\tau_2+t)J_2(\tau_2+t)J_1(t) \quad \text{(A-30)}$$

in which it can be shown that

$$J_2(\tau_1+\tau_2+t) = \left[\frac{1+x^c cQ(\tau_2+t)}{1+x^c cQ(\tau_1+\tau_2+t)}\right]^{\frac{1}{c}+1} \quad \text{(A-31)}$$

and $y_1(\tau_1+\tau_2+t)$ is given by Eq. (A-22) in which the argument τ_1+t is replaced by $\tau_1+\tau_2+t$.

At the end of the third service interval $T_3=T_2+\tau_3$, i.e., $t=\tau_3$, Eq. (A-30) becomes

$$f_{a(T_3)}(x) = G(2)f_{a(0)}[y_1(\tau_3)]J_1(\tau_3) + G(1)F_D^*[y_1(\tau_3)]f_{a(0)}$$

$$[y_1(\tau_2+\tau_3)]J_2(\tau_2+\tau_3)J_1(\tau_3) + F_D^*[y_1(\tau_3)]F_D^*[y_1(\tau_2+\tau_3]$$

$$f_{a(0)}[y_1(\tau_1+\tau_2+\tau_3)]J_2(\tau_1+\tau_2+\tau_3)J_2(\tau_2+\tau_3)J_1(\tau_3)$$

$$\text{(A-32)}$$

The probability of detecting a crack in the size range (x,x+dx) at the third inspection is

$$g_3(x)dx = F_D(x)f_{a(T_3)}(x)dx \quad \text{(A-33)}$$

and the probability of detecting a crack of any size at the third inspection is

$$G(3) = \int_0^\infty g_3(x)dx = \int_0^\infty F_D(x)f_{a(T_3)}(x)dx \quad \text{(A-34)}$$

in which $f_{a(T_3)}(x)$ is given by Eq. (A-32).

(4) jth Service Interval (T_j, T_{j+1})

In a similar fashion, the general expressions for the probability density function of the crack size $a(T_j+t)$ in the jth service interval where $t \leq T_{j+1}-T_j$ $(j \geq 1)$ can be shown as

$$f_{a(T_j+t)}(x) = A_{j+1}(t;j) + \sum_{k=1}^{j} G(j+1-k) A_k(t;j) \quad \text{for } j=1,2,\ldots$$

(A-35)

in which

$$A_k(t;j) = \overbrace{\left\{ \prod_{m=1}^{k-1} F_D^*[y_m(t;j)] \right\}}^{*} f_{a(0)}[y_k(t;j)] \left\{ \prod_{m=1}^{k} J_m(t;j) \right\} \quad (A-36)$$

$$y_1(t;j) = y_1(t) = \frac{x}{(1+x^c cQt)^{1/c}} = W(x,t)$$

$$y_m(t;j) = y_1\left[t + \sum_{n=j-m+2}^{j} \tau_n \right] = \frac{x}{\left\{ 1+x^c cQ\left[t + \sum_{n=j-m+2}^{j} \tau_n \right] \right\}^{1/c}}$$

$$= W\left[x, \ t + \sum_{n=j-m+2}^{j} \tau_n \right]$$

(A-37)

$$J_1(t;j) = J_1(t) = \frac{dy_1(t)}{dx} = \frac{1}{\left[1+x^c cQt \right]^{\frac{1}{c}+1}}$$

$$J_2(t;j) = \left\{ \frac{1+x^c cQt}{1+x^c cQ(t+\tau_j)} \right\}^{\frac{1}{c}+1}$$

(A-38)

$$J_m(t;j) = \left\{ \frac{1+x^c cQ\left[t + \sum_{n=j-m+3}^{j} \tau_n \right]}{1+x^c cQ\left[t + \sum_{n=j-m+2}^{j} \tau_n \right]} \right\}^{\frac{1}{c}+1} \ ; \quad m=3,4,\ldots$$

* Term Equal to 1.0 for k = 1

89

It can be easily shown from Eq. (A-38) that

$$I_k(t;j) = \prod_{m=1}^{k} J_m(t;j) = \cfrac{1}{\left\{ 1+x^c cQ \left[t + \displaystyle\sum_{n=j-k+2}^{j} \tau_n \right] \right\}^{\frac{1}{c}+1}} \left. \right\}$$

$$I_1(t;j) = J_1(t;j) \qquad\qquad\qquad\qquad\qquad\qquad \left. \right\} \quad \text{(A-39)}$$

Consequently, Eq. (A-36) can be simplified as

$$A_1(t;j) = f_{a(o)}[y_1(t;j)] I_1(t;j)$$

$$A_k(t;j) = \left\{ \prod_{m=1}^{k-1} F_D^*[y_m(t;j)] \right\} f_{a(0)}[y_k(t;j)] I_k(t;j) \quad \text{(A-40)}$$

in which $I_k(t;j)$ is given by Eq. (A-39) and $y_m(t,j)$ is given by Eq. (A-37).

(B) GENERAL CASE

For the general situation, the crack growth rate represented by Eq. (8) may not be applicable because of the crack geometry and other factors, especially in the large crack size regime. As a result, a general crack growth computer program, such as CGR, should be used to generate the crack growth master curve. Then, the crack growth damage accumulation can be expressed by Eqs. (4-1) or (4-2) (also see Eqs. (A-1) and (A-2), i.e.,

$$a(t_1) = W[a(t_2), t_2-t_1] \qquad\qquad \text{(A-41)}$$

or

$$a(t_2) = W^{-1}[a(t_1), t_2-t_1] \qquad\qquad \text{(A-42)}$$

in which $t_2 \geq t_1$.

For $t_1=0$ and $t_2=t$, Eqs. (A-41) and (A-42) becomes

$$a(0) = W[a(t), t] \qquad\qquad \text{(A-43)}$$

$$a(t) = W^{-1}[a(0), t] \qquad\qquad \text{(A-44)}$$

Now, Eqs. (A-41) to (A-44) will serve as the transfer functions to transform the statistical distribution of the initial flaw size a(0) into that of the flaw size at any service time.

(1) In the First Service Interval $(0, T_1)$

Following the same derivations presented in Eqs. (A-5) to (A-9), one can easily show that $p(i,\tau)$ is given by Eq. (35) or Eqs. (A-8) and (A-9) in which

$$y_1(\tau) = W(x_1, \tau) \qquad\qquad \text{(A-45)}$$

The probability density function $f_{a(t)}(x)$ of the crack size at t flight hours is obtained from $f_{a(0)}(x)$ of the EIFS through the transformation of Eq. (A-43)(see Eq. (A-10)) as

$$f_{a(t)}(x) = f_{a(0)}[y_1(t)]I_1(t) \qquad\qquad \text{(A-46)}$$

in which $f_{a(0)}(x)$ is given by Eq. (39) and

$$\left. \begin{aligned} y_1(t) &= W(x,t) \\[2ex] I_1(t) &= \frac{dW(x,t)}{dx} = \left[\frac{dW(x,t)}{dt}\right] \bigg/ \left[\frac{dx}{dt}\right] \end{aligned} \right\} \qquad \text{(A-47)}$$

Note that Eqs. (A-46) and (A-47) are identical to Eqs. (41) and (42), respectively.

(2) In the j+1 th Service Interval (T_j, T_{j+1})

Using the transfer functions given by Eqs. (A-41) to (A-44) instead of Eqs. (A-1) to (A-4) (for the special case) and following exactly the same procedure derived above for the special case, one can show that the probability density function $f_{a(T_j+t)}(x)$ of the crack size $a(T_j+t)$ at the service time T_j+t in the j+1 th service interval is given by Eqs. (43) to (47) in the text.

The solutions given by Eqs. (43) to (47) can also be obtained without going through the derivation described above using the following rationale.

Referring to Fig. (21), the probability density function at T_j+t flight hours in the j+1 th service interval consists of j different populations. This is because an additional new population is introduced after each inspection and repair. The first term of Eq. (43) ,

$$A_{j+1}(t) = \left\{ \prod_{m=1}^{j} F_D^*[y_m(t;j)] \right\} f_{a(0)}[y_{j+1}(t;j)]I_{j+1}(t;j) \qquad \text{(A-48)}$$

is the contribution from the original population starting at the zero service time. The term $f_{a(0)}[y_{j+1}(t;j)]I_{j+1}(t;j)$ is the probability density function of the flaw size $a(T_j+t)$ directly obtained from that of $a(0)$ through the transformation of Eq. (A-43) in which t is replaced by T_j+t. However, the original population is subjected to j times of inspections and repairs. Only the fraction of this population which is not detected during each inspection has a contribution to the density function of $a(T_j+t)$. The probability (or fraction) of not being detected during j times of inspections is given by $\left\{ \prod_{m=1}^{j} F_D^*[y_m(t;j)] \right\}$, in which a crack size x at the m th inspection corresponds to a crack size $y_m(t;j)$ prior to service (see Eq. (45)).

The last term of Eq.(43) for k=j, i.e., $G(1)A_j(t;j)$ represents the contribution from the new population introduced after the first repair performed at T_1 (see Fig.(1)) with probability G(1). Similarly, the second term of Eq. (43) for k=1

$$G(j)A_1(t;j) \qquad \text{(A-49)}$$

represents the contribution from the new population introduced after the j th repair performed at T_j. The fraction of cracked details repaired at T_j is G(j). In Eq. (A-49), $A_1(t;j)$ is given by Eqs. (41) and (42) which indicates the density function of the crack size at T_j+t when its crack size at T_j is $a(0)$ (cracks repaired at T_j). $A_1(t;j)$ is obtained from the probability density function of $a(0)$ through the transformation of Eq. (A-41) with $t_1=T_j$ and $t_2=T_j+t$, i.e.,

$$a(T_j) = W[a(T_j+t), t] \qquad \text{(A-50)}$$

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93

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